

Group assignment

# Rubbish Bin Placement and Pickup Scheduling at the University of Melbourne

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## **Abstract**

The location and management of rubbish bins is an important factor in maintaining an environmentally safe, healthy and pleasant campus. Bins should be conveniently placed where they capture the most rubbish whilst maintaining aesthetic and practical considerations. The location of the bins should also be such so that the emptying of the bins can be done in a cost effective manner. We study this problem for the University of Melbourne Parkville Campus. We describe the background to the problem, the model we chose to use, our data collection strategy, the challenges with this, and finally our results and conclusions.

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# 1 Introduction

The client we selected was the University of Melbourne, and our client contacts were Judith Alcorn (Waste Project Officer) and Peter Wuelfert (Operations Manager, SERCO).

The University of Melbourne was established in 1853 and is one of the oldest Universities in Australia. In 2009, there were 35,909 students enrolled and 7,325 full time staff (or equivalent) [MEL09]. The University has a number of campus locations with Parkville being the largest. SERCO is an international service company and their work ranges from the management of facilities, projects and IT systems. This includes the financing, design and build of new facilities, including hospitals and transport systems, as well as their day-to-day operation. They run scientific establishments, and provide critical information to manage traffic, maintain buildings and operate railways. SERCO provides open space maintenance services to Melbourne City Council as well as the University of Melbourne [SER11].

The activity we seek to improve is the placement of rubbish bins on campus, the schedule of pickups from the bins and the routing of the vehicle performing the pickup. The location and management of rubbish bins is an important factor in maintaining an environmentally safe, healthy and pleasant campus. Bins should be conveniently placed where they capture the most rubbish whilst maintaining aesthetic and practical considerations. The location of the bins should also be such so that the emptying of the bins can be done in a cost effective manner. For pragmatic reasons, we chose to handle this as two optimisation problems: first, bin location and pickup scheduling and, second, optimising the route given a pickup schedule. As well as giving an easier problem to model, it also reflects the real life aspect that the actual emptying of the bins is done as part of other activity by the ground staff and so is only loosely tied to the actual locating of the bins. If we were to apply this model to other problems such as location of ATM (i.e. cash machines), where the routing of the security personal needs to be more carefully controlled, then a tighter integration between the two problems would be needed.

We considered only the external bins used by the general campus population and were not concerned with the location of the larger skip type bins nor bins within buildings. Also, the rubbish we included in our study was generally food container waste that we divided into landfill waste and recyclable waste. We ignored hazardous waste or 'hard rubbish'. We aimed to minimise the 'inconvenience cost' of putting rubbish into bins. For the routing problem, we aimed to minimise total route length

for each pickup whilst meeting the required pickup schedule.

In devising the model, we considered a number of factors arising from our own observations and assumptions as well as input from discussions we had with Judith Alcorn and Peter Wuelfert. One of the first tasks was to devise a network model to provide a framework to the problem. The network model makes use of map data of the Parkville campus and we developed a program in XpressMP [FIC09] to present this map. The program is also a data processing platform to process our raw survey data into the input parameters to the model. Edges in the network model are roads or pathways and vertices in the model are intersections of roads or pathways on campus. We gathered traffic data and developed a traffic model across the network to assist with generating the parameters for the model. Other data we gathered was data on seating areas, shop locations and the locations where rubbish from the pickups was dropped off.

Our results showed a reduction by over a third in the “inconvenience cost”. The optimisation provides a solution that gives us locations for 40 bin locations of which 24 are not currently used. Currently, 37 bin locations are used.

In the course of the project, and in particular the data gathering phase, we determined that there is likely to be a large variability to the traffic and food consumption patterns on campus. In particular, we were told by Peter that different weather conditions lead to different patterns in rubbish generation. Other sources of variability are:

1. People’s choice of coffee and lunch purchases
2. People’s walking patterns

The remainder of this report is divided up as follows: Chapter 2 presents the problem background and considers the factors influencing the location of bins and scheduling of pickup. Chapter 3 presents the model as a mathematical model, Chapter 4 discusses our data collection process, Chapter 5 presents the results and finally Chapter 6 presents the conclusions.

We would like to thank Judith Alcorn and Peter Wuelfert for their assistance with this project and taking the time to answer our questions and to meet with us.

## 2 Proposed Improvements

This chapter discusses in a general context the bin location and scheduling problem introduced above and the factors we did and did not model. We seek to determine if an improvement can be made to the location of bins on campus and to provide advice on the pickup schedule and collection routing. One aspect that we were careful about was to make sure the model was such that we did not obtain extreme solutions such as having a bin everywhere, having only a few bins at key locations or emptying the bin every hour. In subsequent sections we discuss rubbish and how it is generated, factors influencing bin placement and the rubbish pickup process.

### 2.1 Rubbish and Rubbish Generation

The rubbish we considered was the rubbish arising from the purchase of coffee, other beverages and take away food. The source of these items are food outlets on campus. We distinguished between two types of rubbish - waste and recyclable.

In the work, we used the idea of rubbish generation: Rubbish is generated at the point where it is ready to be placed into a bin. For example, the location where a coffee cup is finished with is the point where the rubbish is generated. The location where coffee or lunch was purchased is not necessarily the point where the rubbish was generated. We make this distinction as people will often take their lunch to a seating area to consume or take their coffee with them on the way to a lecture.

We term coffee and beverage containers to be ‘walking rubbish’ and take away food ‘seating rubbish’. We make this distinction because the generation of these types of rubbish is a little different: Walking rubbish arises from items that can be consumed on the move and will spread out from the point of purchase. Seating rubbish arises from items like take-away food and is consumed at seating areas. When the meal is completed people prefer to place this as soon as possible in the nearest bin.

We make the following assumptions about behaviour: First, people prefer to have a bin that is on the way to wherever they are going. If someone finishes with a coffee cup they prefer a bin that is along the route they will be going to their next lecture or other activity. Second, people will, in general, do the right thing and will not drop rubbish immediately they don’t find a convenient bin; they might be annoyed that there is not a bin nearby but they will make an effort to find a bin. Only if they

can not find a bin after a time will they drop the rubbish on the ground. To model the idea of rubbish generation locations along with our assumptions about people's behaviour, we use a traffic model and the idea that edges in the network with low traffic have a high service cost when considering how rubbish travels from points where it is generated to a bin. Service cost represents the inconvenience factor of taking rubbish from one vertex to another. Terming this an inconvenience is perhaps making it a little unclear as strictly speaking inconvenience would just be the length of the edge however by introducing the actual flow of the traffic we are introducing a likelihood factor as well - rubbish is more likely to travel along an edge based on the traffic along the edge and the length of the edge. We turn this around to turn it into a cost to make the model simpler to understand and work with mathematically.

## 2.2 Bin Location

This section discusses in general factors that influence bin placement. These are a combination of interview input and our own observations and assumptions.

There are a number of factors to consider when placing rubbish bins:

1. The bin needs to be placed at a location where it is effective in capturing rubbish. These will be in high traffic areas that are close to where rubbish is generated.
2. The bin needs to be easy to access. There is no point in placing a bin on a little used pathway or tucked away behind some building even if it is close 'as the crow flies' to a high traffic or rubbish generation area.
3. There are aesthetic considerations when placing bins. Placement of bins outside historic buildings or in highly visible locations is discouraged. Placement of bins should be balanced between discrete and accessible.
4. Bins should be placed so that they can be emptied conveniently.

The bins we considered are external bins for the use of general campus population for placement of rubbish. They are placed along walk ways and around seating areas. We did not consider the large rubbish bins located outside buildings nor general use rubbish bins inside buildings. We ignored constraints on bin placements due to aesthetic reasons although this would be fairly easy to include in the model: we would assign a cost to placing a bin at a location (or exclude the location entirely as one where a bin could not be placed).

For placement, we only permitted placement at vertices of the network model. In reality bins can be placed along edges however our edges are relatively short and placement of a bin at one end of an edge rather than along an edge is a reasonable approximation.

## 2.3 Rubbish Pickup

Rubbish bin collection around campus is undertaken by the SERCO ground staff who also perform other activities throughout such as leaf collection. Rubbish from bins is collected using a large golf-cart type buggy and taken to a number of locations where there are large skip-type bins. The cart can take rubbish from 12 full bins (giving a capacity of 1440L) and is parked over night in the South Lawn Underground Car Park. The cart is also used for other activities during the day and parked somewhere on campus whilst doing other things (for example collecting litter on ground). Although there are multiple carts used by grounds staff, only one of these has the required compartment to carry rubbish bags.

The pickup schedule is governed at a high level by the contract SERCO have with the University. For high profile areas, bins need to be emptied at least twice a day. These areas are: South Law, Professor's Walk, Concrete Lawn, The Old Quad, In front of Wilson Hall. The bins in those places have to be emptied the first time at 9am (note that this constraint was not included in the model). In other places, the bins have to be emptied at least once a day. The same staff collect litter lying around (on the ground) when they see some. There is a one person assigned to collect the rubbish out of a team of eight people and at least one person has to be on campus for four hours a day. Sometimes there is a need to collect rubbish on Saturday and/or Sunday as well special events. Given that the rubbish bin collection has some component of flexibility and is included in other daily activities, we decided that the routing would be included as 'weak' constraints in the location model. In other words, we incorporated only enough constraints to ensure that routing would not be unacceptably expensive.

The following are some of the considerations:

1. Bins are emptied enough times to ensure bin does not become full / overflow.
2. Bins are not emptied unnecessarily.
3. Fuel and servicing, capital costs and labour costs.

For our problem we included constraints 1 and 2. We modelled per trip cost based on distance travelled but ignore fixed costs for vehicles since the vehicles are still needed for other purposes.

Figure 2.1 shows a representative fragment of the network model we used. Vertices are shown with their rubbish generation counts for a single time period. Along each edge we show the traffic for the same time period in both directions. The nodes marked with an  $S$  are seating nodes.

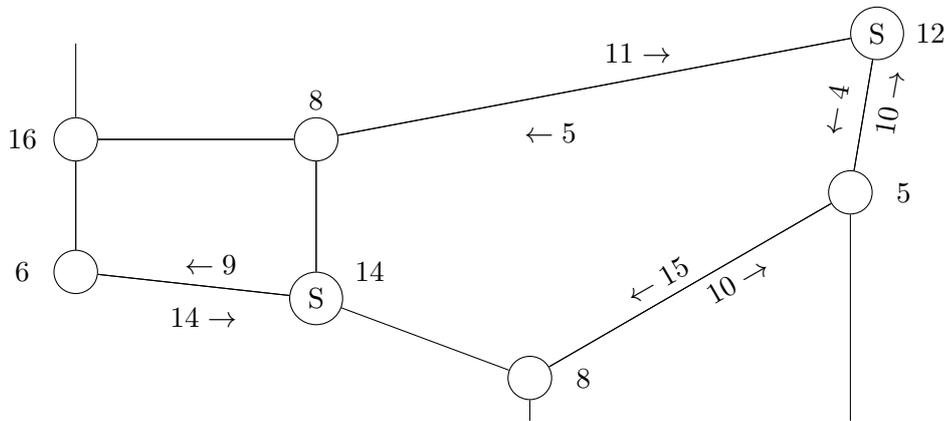


Figure 2.1: Network Fragment Showing Various Parameters

The following chapter details the mathematical model that we developed based on the above discussions.

## 3 Solution Methodology

### 3.1 Overview

This section outlines our approach from a high level. The following sections will describe the components of the solution in more detail.

Figure 3.1 illustrates the structure of the problem, showing the main relationships between components. The input to the problem is demand, in the form of rubbish to be put in a bin. This demand, together with the chosen locations and the pickups scheduled, will determine the amount of rubbish in each bin over time. Both bin location and collection scheduling also affect the routing problem. The best route between a set of bins obviously depends on the locations of the bins, and the collection schedule determines which bins will be collected during the same time interval, and therefore which bins should be visited on the same route. The optimisation objectives appear in the bottom of the figure. These are convenience, which is a direct result of which bin has space to receive which rubbish, and collection cost, which depends on the number and length of collection routes.

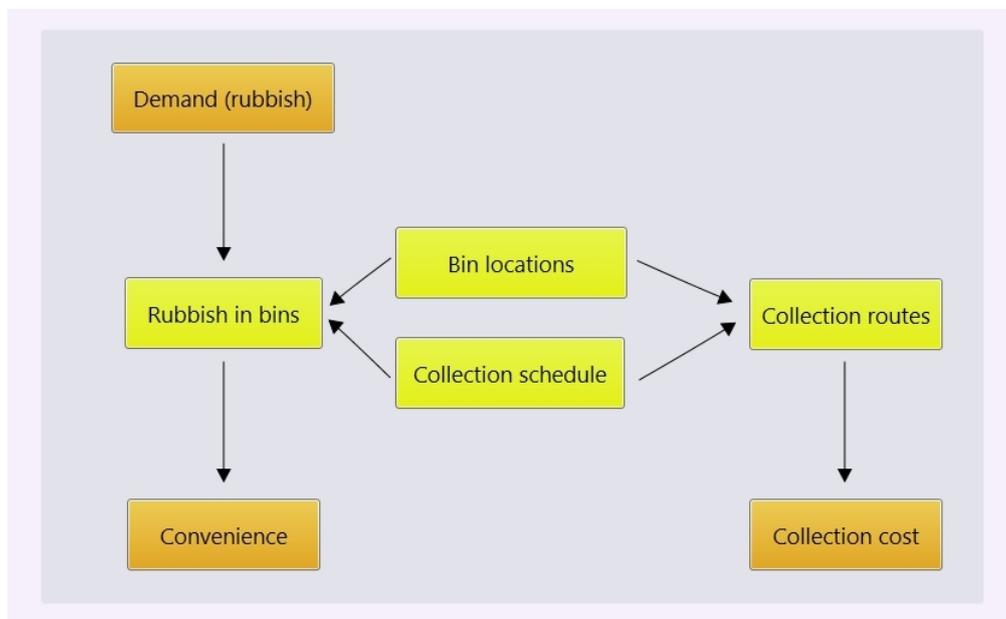


Figure 3.1: Problem structure and interactions

It is important to note that the locations of the bins interact with each other. We assume that people will put rubbish in the most convenient bin that is not full. Therefore the amount of rubbish put into a particular bin depends not only on the location of that bin, but also on the locations of other nearby bins, and the amount of rubbish currently in those bins. If a different location is chosen for bin  $i$ , another bin may become the preferred bin for rubbish that would previously have been put in bin  $i$ . This means that the location of one bin cannot be considered in isolation from the locations of all of the other bins. Furthermore, the amount of rubbish in a bin, and therefore its availability to accept more rubbish, depends on when the last pickup was scheduled for that location. If the scheduled pickup time for a bin location changes, that bin may be able to accept more or less rubbish at this time, which will have an impact on the volume of rubbish in other bins nearby.

The two objectives identified above clearly compete with one another. The most convenient solution will have a high collection cost, and the least expensive solution is likely to be very inconvenient. We considered weighting the objectives, but decided instead to prioritise convenience, while limiting the collection cost to an acceptable level. We had several reasons for this decision. Firstly, as convenience is not easily valued in terms of money, we felt that any weighting we invented would be fairly arbitrary. Secondly, from discussions with our University of Melbourne contact, we believe the university is currently most concerned with improving the usage of bins (especially recycling bins), and keeping the campus presentable, rather than reducing cost. We felt that prioritising convenience would be more in line with these goals, as more convenient bin locations should lead to better bin usage and less litter on the ground. Finally, given the current arrangement with SERCO, there is not much scope for optimisation of collection routes. Rubbish collection is not typically performed as a separate task but rather mixed in with other tasks of the ground staff. This makes it difficult to accurately reflect the real cost of rubbish collection, which we felt was a good reason to avoid using that cost as our main objective.

An attempt to solve the entire problem using one monolithic model confirmed our expectation that the problem is too large to be handled this way. We therefore decided to split the model into two pieces. Our chosen approach is illustrated in Figure 3.2. The decision to prioritise convenience, and the tight coupling of the bin location and pickup scheduling problems, made the routing problem an obvious choice to be taken out and treated separately. Our approach was to first tackle the bin location and pickup scheduling problems together, aiming to maximise convenience, and then to feed the results into a set of routing problems, one for each time period during which any bins were scheduled for collection. Each routing problem had as input a set of bins to be visited, with each bin having a specified location and volume of rubbish to be collected.

It is worth noting that the routing results are unlikely to be put into practice. Our

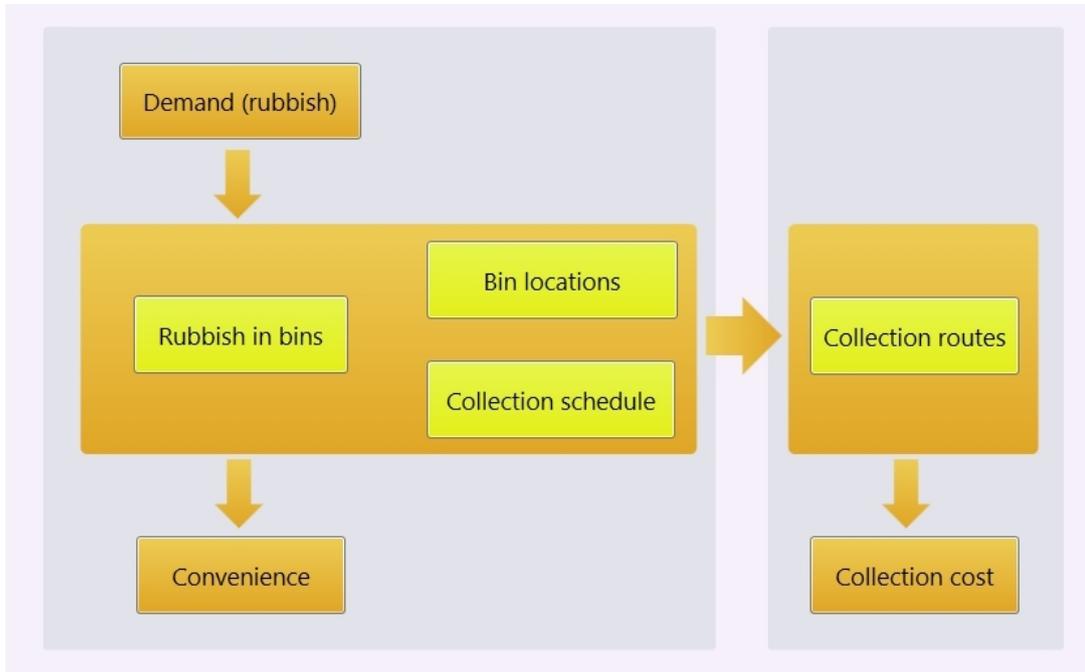


Figure 3.2: Our approach to the problem

model is only a very rough approximation to the way bins are collected in reality. As mentioned above, grounds staff tend to use the vehicle for other purposes during the day as well as collecting bins, and while collecting bins they will often stop to perform tasks such as picking up litter on the ground. Furthermore, because the campus is quite a small area, optimal routes would not necessarily be a significant improvement over the current process. Simple heuristics naturally employed by the driver such as "I'm driving past, so I may as well pick up this bin now," and "while I'm over this side of the campus I'll get that bin so I don't have to come back later," may well be sufficient to produce reasonable routes. However, we decided to go through the routing process anyway, because although the routes produced are unlikely to be actually used, they do serve to demonstrate the feasibility of our bin location and pickup scheduling solution, and may be helpful as a guide for contract negotiation between the University of Melbourne and SERCO.

## 3.2 Bin Location and Collection Scheduling

Our objective for the bin location and collection scheduling subproblem was to maximise convenience. We achieved this by assigning an 'inconvenience cost' of putting rubbish generated in a particular location into a bin at another location, and minimising the total cost incurred by the solution.

The mathematical program we used is shown below, and the key points are discussed here.

The model uses a network of nodes, some of which generate rubbish (the set  $\mathbb{L}^R$ ), and some of which are potential bin locations (the set  $\mathbb{L}^B$ ). Most nodes in our data belonged to both sets, but some nodes (for example nodes on lawns) are not potential bin locations.

We considered two bin/rubbish types; recycling and general waste. We assume that rubbish of a particular type can only be put in a bin of that same type.

The bin size in litres is given, and there is a limit on the number of bins of the same type which may be placed in the same location. We used a value of 2 for this limit, as we observed that most current bin locations have one bin of each type, but some have an extra bin of one or the other type.

We divided time into discrete intervals, considering one full day with 10 time periods. The time periods correspond to roughly one hour each, but we excluded the night time hours as there is not much rubbish generated at night, and we also don't want to schedule collections at those times. The amount of rubbish in a bin after the last time interval carries over to the first time interval. Since we are only considering one day at a time, this wrapping around enforces at least one collection of every bin every day. This restriction is not a problem as it is already university policy to empty bins every day. However, limiting the time period to one day does limit the flexibility of the solution; we are assuming that the same collection schedule will be used every day, and therefore it isn't possible, for example, to visit a bin location for collection an average of 1.5 times per day.

Our main variables were a set of real variables  $x_{ijkt}$  specifying the volume of rubbish of type  $k$  generated at location  $i$  put in a bin at location  $j$  during time  $t$ , and a set of binary variables  $y_{jt}$  indicating whether or not a pickup was scheduled for bins at location  $j$  at the end of time interval  $t$ .

We also had two sets of variables  $r_{jkt}^B$  and  $r_{jkt}^A$  specifying the volume of rubbish of type  $k$  present in bins at location  $j$  after time interval  $t$ , respectively immediately before and immediately after the scheduled pickups for that time period are carried out.

Standard demand and capacity constraints (constraints 3.1 and 3.2) were used to ensure that all rubbish was put in a bin, and that no bin location had more than the maximum allowed volume of rubbish at any time. The actual number of bins assigned to a location was calculated afterwards by finding the maximum volume of rubbish at the location over all times, and choosing the smallest number of bins to meet this requirement.

Constraints 3.3, 3.4, 3.5 and 3.6 enforce consistency of the volume of rubbish at bin locations over time. The volume of rubbish in a bin at a particular time immediately

before rubbish collection is calculated by adding the amount of rubbish put in the bin during this time step to the rubbish already there after rubbish collection in the previous time step. The volume of rubbish at a bin location immediately after rubbish collection is 0 if a pickup is scheduled for this location at this time, otherwise it is the same as the volume of rubbish before rubbish collection.

In order to limit the collection cost resulting from a solution to this part of the problem, we added two extra constraints on the pickups scheduled.

- Constraint 3.7 limits the total number of pickups scheduled across the day. This prevents a solution with excessive bin collection, and also serves to limit the total number of bins, as all locations with any rubbish must have at least one pickup scheduled.
- Constraints 3.8 and 3.9 put a restriction on the number of time intervals during which pickups may be scheduled, to prevent the spreading out of pickups across the day, which is inefficient when it comes to routing. We set the limit at 4 collection routes per day, as currently the grounds staff are required to be on the campus for four hours each day.

Let

$\mathbb{L}^B$	be the set of potential bin locations
$\mathbb{L}^R$	be the set of locations at which rubbish is generated
$\mathbb{B}$	be the set of bin/rubbish types
$S$	be the capacity of a bin in litres
$N$	be the maximum number of bins of the same type allowed to be placed in the same location
$P$	be the maximum number of pickups to be scheduled in total
$Q$	be the maximum number of times during which pickups are scheduled
$C_{ij}$	be the cost of using bin $j$ for rubbish generated at location $i$
$R_{ikt}$	be the amount of rubbish of type $k$ generated at location $i$ during time interval $t$ (in litres)
$A$	be the first time period under consideration
$Z$	be the last time period under consideration
$\mathbb{T} = \{A \cdots Z\}$	be the set of time periods under consideration

Let

$x_{ijkt}$	be the volume of rubbish of type $k$ generated at location $i$ which is put in a bin at location $j$ during time $t$
$r_{jkt}^B$	be the volume of rubbish of type $k$ in bins at location $j$ at time $t$ , immediately before rubbish is collected
$r_{jkt}^A$	be the volume of rubbish of type $k$ in bins at location $j$ at time $t$ , immediately after rubbish is collected
$y_{jt}$	$= \begin{cases} 1 & \text{if a pickup is scheduled for bins at location } j \text{ at time } t \\ 0 & \text{otherwise} \end{cases}$
$p_t$	$= \begin{cases} 1 & \text{if } t \text{ has been selected as a time to schedule pickups} \\ 0 & \text{otherwise} \end{cases}$

Figure 3.3: Parameter and decision variable definitions for bin location and collection schedule subproblem.

minimise  $\sum_{i \in \mathbb{L}^R} \sum_{j \in \mathbb{L}^B} \sum_{k \in \mathbb{B}} \sum_{t \in \mathbb{T}} C_{ij} x_{ijkt}$   
s.t.

$$\sum_{j \in \mathbb{L}^B} x_{ijkt} \geq R_{ikt} \quad \forall i \in \mathbb{L}^R, t \in \mathbb{T}, k \in \mathbb{B} \quad (3.1)$$

$$r_{jkt}^B \leq NS \quad \forall j \in \mathbb{L}^B, k \in \mathbb{B}, t \in \mathbb{T} \quad (3.2)$$

$$r_{jkt}^B \geq r_{jkt-1}^A + \sum_{i \in \mathbb{L}^R} x_{ijkt-1} \quad \forall j \in \mathbb{L}^B, k \in \mathbb{B}, t \in \mathbb{T} : t > A \quad (3.3)$$

$$r_{jkt}^A \geq r_{jkt}^B - NS y_{jt-1} \quad \forall j \in \mathbb{L}^B, k \in \mathbb{B}, t \in \mathbb{T} : t > A \quad (3.4)$$

$$r_{jkt}^B \geq r_{jkt}^A + \sum_{i \in \mathbb{L}^R} x_{ijkz} \quad \forall j \in \mathbb{L}^B, k \in \mathbb{B}, t = A \quad (3.5)$$

$$r_{jkt}^A \geq r_{jkt}^B - NS y_{jz} \quad \forall j \in \mathbb{L}^B, k \in \mathbb{B}, t = A \quad (3.6)$$

$$\sum_{j \in \mathbb{L}^B} \sum_{t \in \mathbb{T}} y_{jt} \leq P \quad (3.7)$$

$$\sum_{t \in \mathbb{T}} p_t \leq Q \quad (3.8)$$

$$y_{jt} \leq p_t \quad \forall t \in \mathbb{T}, j \in \mathbb{L}^B \quad (3.9)$$

$$x_{ijkt} \geq 0 \quad \forall i \in \mathbb{L}^R, j \in \mathbb{L}^B, k \in \mathbb{B}, t \in \mathbb{T} \quad (3.10)$$

$$r_{jkt}^B \geq 0 \quad \forall j \in \mathbb{L}^B, k \in \mathbb{B}, t \in \mathbb{T} \quad (3.11)$$

$$r_{jkt}^A \geq 0 \quad \forall j \in \mathbb{L}^B, k \in \mathbb{B}, t \in \mathbb{T} \quad (3.12)$$

$$y_{jt} \in \{0, 1\} \quad \forall j \in \mathbb{L}^B, t \in \mathbb{T} \quad (3.13)$$

$$p_t \in \{0, 1\} \quad \forall t \in \mathbb{T} \quad (3.14)$$

Figure 3.4: Mathematical model for bin location and collection schedule subproblem.

### 3.3 Collection Routing

For the routing problem the input is a set of bins to be visited, each bin having a location and a volume of rubbish to be collected. Our objective is to minimise the route length.<sup>1</sup> Our final mathematical formulation is included below.

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<sup>1</sup>We refer to [XAP07], pp. 148 for the initial attempt at a model, since we refined our approach from there.

The routing network has vertices of several different types. There is a set of depots  $\mathbb{D}$ , a set of bins  $\mathbb{B}$ , and a vertex  $P$  representing the car park. The set of all vertices is referred to as  $\mathbb{A}$ .

Only one vehicle is used because only one of the grounds maintenance carts has the correct compartment to carry rubbish bags. The vehicle starts at the car park vertex, then travels to a bin. From a bin the vehicle may travel to another bin, or to a depot, where it will drop off all rubbish collected so far. From a depot, the vehicle may visit another bin, or return to the car park, ending the route.

We used binary variables  $x_{ij}$  indicating whether or not the edge<sup>2</sup>, from vertex  $i$  to vertex  $j$  was used in the solution. We also had variables  $y_{ij}$  indicating the load of the vehicle on the edge from vertex  $i$  to vertex  $j$ . Note that although some depots may be visited more than once, no edge can be used more than once, because no edges are allowed between depots.

Constraints 3.15, 3.16 and 3.17 were used to eliminate edges from the solution as follows.

- Loops, where both ends of an edge are the same vertex, are not allowed.
- Edges leading from the car park to a depot are not allowed as the car is empty so there is no benefit in going to the depot.
- Edges leading from a bin to the car park are not allowed as the vehicle should be parked empty, so it needs to visit a depot immediately before parking.

We assume that when a bin is visited all rubbish is collected. Therefore, each bin should be visited exactly once (constraints 3.18 and 3.19).

The vehicle must leave the car park on exactly one edge leading to a bin (constraint 3.20) and return on exactly one edge from a depot (constraint 3.21).

A depot may be visited multiple times or not at all, as long as the number of edges in is the same as the number of edges out (constraint 3.22).

Constraint 3.24 ensures that when a bin vertex is visited, the rubbish to be collected is added to the load of the vehicle when it leaves. The load on the vehicle is restricted to its capacity using constraint 3.23.

At a depot vertex the load when leaving is always 0 because all of the rubbish should have been dropped off (constraint 3.25). Note that we assume infinite capacity of the skip bins at depots.

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<sup>2</sup>Note that the network used here is different from the network discussed in the previous section. For the collection routing the network involves primarily just the bins. The edge lengths are precalculated by the BinNetwork program using shortest path between the bins.

Let

$V$	be the vehicle capacity in litres
$P$	be the location of the car park
$\mathbb{D}$	be the set of depots (rubbish drop off locations)
$\mathbb{B}$	be the set of bins to be visited
$\mathbb{A} = \mathbb{B} \cup \mathbb{D} \cup \{P\}$	be the set of all vertices in the network
$D_{ij}, i, j \in \mathbb{A}$	be the length of the shortest path from vertex $i$ to vertex $j$
$R_i, i \in \mathbb{B}$	be the litres of rubbish to be collected from each bin

Let

$x_{ij} =$	$\begin{cases} 1 & \text{if the edge from vertex } i \text{ to vertex } j \text{ is used in the solution} \\ 0 & \text{otherwise} \end{cases}$
$y_{ij}$	be the load in litres of the vehicle when it travels along the edge from vertex $i$ to vertex $j$

Figure 3.5: Parameter and decision variable definitions for routing subproblem.

$$\begin{aligned} & \text{minimise } \sum_{i,j \in \mathbb{A}} D_{ij} x_{ij} \\ & \text{s. t.} \end{aligned}$$

$$x_{ii} = 0 \quad \forall i \in \mathbb{A} \quad (3.15)$$

$$x_{Pi} = 0 \quad \forall i \in \mathbb{D} \quad (3.16)$$

$$x_{iP} = 0 \quad \forall i \in \mathbb{B} \quad (3.17)$$

$$\sum_{i \in \mathbb{A}} x_{ij} = 1 \quad \forall j \in \mathbb{B} \quad (3.18)$$

$$\sum_{i \in \mathbb{A}} x_{ji} = 1 \quad \forall j \in \mathbb{B} \quad (3.19)$$

$$\sum_{i \in \mathbb{B}} x_{Pi} = 1 \quad (3.20)$$

$$\sum_{i \in \mathbb{D}} x_{iP} = 1 \quad (3.21)$$

$$\sum_{i \in \mathbb{A}} x_{ij} = \sum_{i \in \mathbb{A}} x_{ji} \quad \forall j \in \mathbb{D} \quad (3.22)$$

$$y_{ij} \leq V x_{ij} \quad \forall i, j \in \mathbb{A} \quad (3.23)$$

$$\sum_{j \in \mathbb{A}} y_{ij} = \sum_{j \in \mathbb{A}} y_{ji} + R_i \quad \forall i \in \mathbb{B} \quad (3.24)$$

$$y_{ij} = 0 \quad \forall i \in \mathbb{D}, j \in \mathbb{A} \quad (3.25)$$

$$x_{ij} \in \{0, 1\} \quad \forall i, j \in \mathbb{A} \quad (3.26)$$

$$y_{ij} \geq 0 \quad \forall i, j \in \mathbb{A} \quad (3.27)$$

Figure 3.6: Mathematical model for routing subproblem.

This set of constraints does not prevent subtours within our route, despite our use of load variables. The load variables prevent cycles involving only bin vertices, but because we have multiple depots it is still possible to get a nonsensical solution with a subtour involving a set of bins and one or more depots. To combat this we added subtour breaking constraints on the fly, using a very simple algorithm (Algorithm 1). The algorithm runs the problem in a loop, finding the optimal solution allowing subtours, then checking the resulting graph to determine whether or not all bins are connected to the car park. The set of all vertices connected to the car park  $\mathbb{C}$ , is discovered using depth first search. If there is at least one bin not connected to the car park, the algorithm adds a constraint to ensure that at least one edge is included between the set of nodes connected to the car park, and the set of nodes

not connected to the car park. This constraint is correct, since for any subset  $S \subset A$  including a bin and not the car park, a valid route must include at least one edge leading from  $A \setminus S$  to  $S$ .

```

repeat
  solve the optimisation problem;
  addedConstraint := false;
   $C := \emptyset$ ;
  explore ( $P$ );
  if  $\exists b \in B : b \notin C$  then
    add constraint:  $\sum_{i \in C} \sum_{j \in A \setminus C} x_{ij} \geq 1$ ;
    addedConstraint := true;
  end
until addedConstraint = false;

```

**Algorithm 1:** Subtour Elimination

```

Input: A vertex  $v$ 
 $C := C \cup \{v\}$ ;
foreach edge  $(v, u)$  used in the current solution do
  if  $u \notin C$  then
    explore( $u$ );
  end
end

```

**Function** explore( $v$ )

## 4 Data Collection

In the previous chapter we developed a comprehensive model for the problem with a number of components which will allow us to compute an optimised solution for rubbish bin placement at the University of Melbourne and to calculate a pickup schedule for rubbish collection. The next step is to find values for the model parameters. We start by describing the different data types required and discuss how we collected them. Due to the high amount of data needed and the limited time available, there were a number of problems and issues, which we discuss.

### 4.1 Required Data

As mentioned in Section 2.1, we ignored special types of rubbish and considered only two different types: recycling rubbish and landfill waste. From observations on campus and after talking to Peter Wuelfert, a reasonable approximation to differentiate these two types is given in the Table 4.1 which we apply right at the end of our calculations.

Waste rubbish	Recycling rubbish
60%	40%

Table 4.1: Distribution of waste and recycling rubbish

#### 4.1.1 Rubbish Generation

As an approach to achieve a reliable data collection we approximate all generated rubbish by considering only two distinct types of rubbish:

- “Walking rubbish”
- “Sitting rubbish”.

“Walking rubbish” is given by items such as coffee cups purchased at one of the coffee outlets and spread out by people taking the cups with them. Therefore, the amount  $A^{\text{walking rubbish}}$  of “walking rubbish” at a particular node  $i$  in the graph

$G = (V, E)$  representing the campus pathways must be related with the amount of people walking along edges connected to the vertex  $i$  and the amount of sold coffees  $S_t$  in the stores at a certain time  $t$ .

Furthermore people on campus might bring their own food or buy a meal or a beverage on campus to eat or drink it in certain spots. Hence “sitting rubbish” means rubbish which is generated by food or beverages in disposable containers at certain seating area vertices, such as nodes near one of the lawns or nodes directly in front of food stores with a seating area. Of course, this rubbish may have originated from a shop further away, but we can ignore this fact. For our work, we assume that “sitting rubbish” is generated directly at a seating area  $j$  and the amount  $A^{\text{sitting rubbish}}$  of this rubbish type produced at a particular node is correlated with the amount of people sitting there at a certain time  $t$ .

In the following we will present a way to approximate  $A^{\text{walking rubbish}}$  and  $A^{\text{sitting rubbish}}$  in order to calculate the total rubbish generation  $R_{it} = A_{it}^{\text{walking rubbish}} + A_{it}^{\text{sitting rubbish}}$  at vertex  $i$  for time period  $t$ .

If  $i$  is a seating area we will calculate:

$$R_{it} = \frac{1}{5} S_t \frac{I_{it}}{\sum_j I_{jt}} + W_{it}^s \quad (4.1)$$

if  $i$  is not a seating area then the calculation is

$$R_{it} = \frac{1}{5} S_t \frac{I_{it}}{\sum_j I_{jt}} \quad (4.2)$$

Where  $I_{it}$  is the traffic count entering vertex  $i$  at time  $t$ ,  $S_t$  is the total coffee count across campus at time  $t$  and  $W_{it}^s$  is the seating count for vertex  $i$  at time  $t$ . Division is by five as we take it that on average there are five rubbish items per litre.

Rubbish generation by type is calculated due to (4.1) i.e.

$$R_{it}^w = 0.6 R_{it} \quad (4.3)$$

$$R_{it}^r = 0.4 R_{it} \quad (4.4)$$

Next we discuss other data we needed and how we obtained it.

### 4.1.2 Service Cost

In this section we want to find a formula to calculate the service cost. The Service cost  $C_{ijt}$  is a value which gives the inconvenience of rubbish created at node  $i$  being put in a bin at node  $j$ . It is proportional to the distance of the nodes, i.e. whenever the node distance of considered nodes increases,  $C_{ijt}$  will increase as well; and vice

versa. Moreover it is inversely proportional to the traffic amount between  $i$  and  $j$ . Whenever the number of people walking from one node to the other is high,  $C_{ijt}$  will be small, and when the number of people is low  $C_{ijt}$  will be large. We compute the service cost with respect to the distances of two nodes and the traffic amount between those nodes.

$C_{ijt}$  is the service cost from vertex  $i$  to vertex  $j$  for time period  $t$ . Vertex  $i$  is from the set of all possible locations where rubbish can be generated and  $j$  is from the set of all potential bin locations.

$$C_{ijt} = \sum_{e \in P(i,j)} \frac{D_e}{T_{et}} \quad (4.5)$$

where  $P(i, j)$  is the set of edges for the shortest path from  $i$  to  $j$ ,  $D_e$  is the length (i.e. distance) along the edge  $e$  and  $T_{et}$  is the traffic along  $e$  at time  $t$ .

## 4.2 Data Collection

With the thoughts from the section above, we can conclude that we need data in three different types:

- People leaving a shop with bought food / beverages
- People sitting at a seating area
- People walking on an edge

the problem is, we need several data values for every node and every edge; and this not only once a day, but for all considered time steps. Since we have 139 vertices with 179 edges and 9 different times of a day this is simply not measurable in a reasonable amount of time. Therefore we extrapolated measured data using several data samples.

### 4.2.1 Data Profiles

Since it would take too much time and effort to measure the data at every node for every considered time period, we decided to build representative data samples in order to extrapolate other single measurements. We proceeded as follows:

At first we did several whole day observations at selected nodes and edges. The “selected” nodes were nodes for food stores which are frequently visited, seating areas with a higher amount of people and edges with many pedestrians. Those were considered to be relatively representative for all the other nodes and paths

09:00	10:00	11:00	12:00	13:00	14:15	15:15	16:15	17:15
People leaving with food - Place 1: Professor's Court Cafeteria								
8	11	16	15	14	13	11	5	0
People sitting - Place 1: Professor's Court Cafeteria								
9	9	8	3	8	17	11	23	13
People walking - Place 1: Node 56 to 59								
25	66	68	90	63	40	60	53	29

Table 4.2: Whole day observations for different data types

of the same type. Furthermore we assume here that food and drink consumption in all seating areas resembles the coffee consumption and the demand in all stores, and that the walking pattern at all edges behaves equally over the day; merely the magnitude differs. To write things more generalized now, we introduce a set (4.6) for the different data types

$$P = \{b, s, w\}, \quad (4.6)$$

where the letters represent data for

- b - people leaving a shop with bought food / beverages
- s - people sitting at a seating area
- w - people walking along an edge.

What we actually did then was to measure the number of people at the chosen nodes for every considered time and for each data type separately. We used an observation time of five minutes for  $b$  and  $w$ , and since there was not much movement at seating areas, for  $s$  we just took a point sample. Some of the resulting data are shown in table (4.2).<sup>1</sup> In order to create the profile we begin by calculating the maximum value (4.7) for every sample  $i \in I = \{1, 2, \dots, n\}$  of type  $p$ .

$$V_{i,max}^p := \max \{V_{it}^p \mid t \in \mathbb{T}\}, \quad (4.7)$$

where  $\mathbb{T}$  is the set of considered time periods.

We then normalize the other values by  $V_{i,max}^p$  according to

$$\tilde{V}_{it}^p = \frac{V_{it}^p}{V_{i,max}^p} \quad \forall t \in \mathbb{T}. \quad (4.8)$$

<sup>1</sup>The other collected data for the profiles can be found in the appendix (7).

Then our general profile for data type  $p$  is given by

$$G_t^p = \frac{\sum_i \tilde{V}_{it}^p}{|I|} \quad \forall t \in \mathbb{T}. \quad (4.9)$$

In fact, we used  $|I| = 2$  for  $p \in \{b, s\}$  and  $|I| = 4$  for  $p = w$  and got the general profiles shown in (4.1).

With (4.9) and just one observation  $W_{t_0}^p$  of a node or edge at a particular time



Figure 4.1: General profiles due to whole day observations

$t_0 \in \mathbb{T}$ , we are able to extrapolate the values for all other time periods using the profile via (4.10).<sup>2</sup>

$$W_t^p = \frac{W_{t_0}^p}{G_{t_0}^p} G_t^p \quad \forall t \in \mathbb{T} \quad (4.10)$$

After calculating all needed data, we can use it as an input for our model and run the optimiser.

<sup>2</sup>The completed data calculation can be found in the files “TrafficData.dat” and “RubbishModelData.dat”.

## 5 Results and Discussion

With the data we obtained from our calculation according to Chapter 4 we ran the program to optimise the problem for 24 minutes. Figure (5.1) shows the branch and bound representation tree and the progress of the MIP search during the optimisation process. Below we present results for both the rubbish bin placement

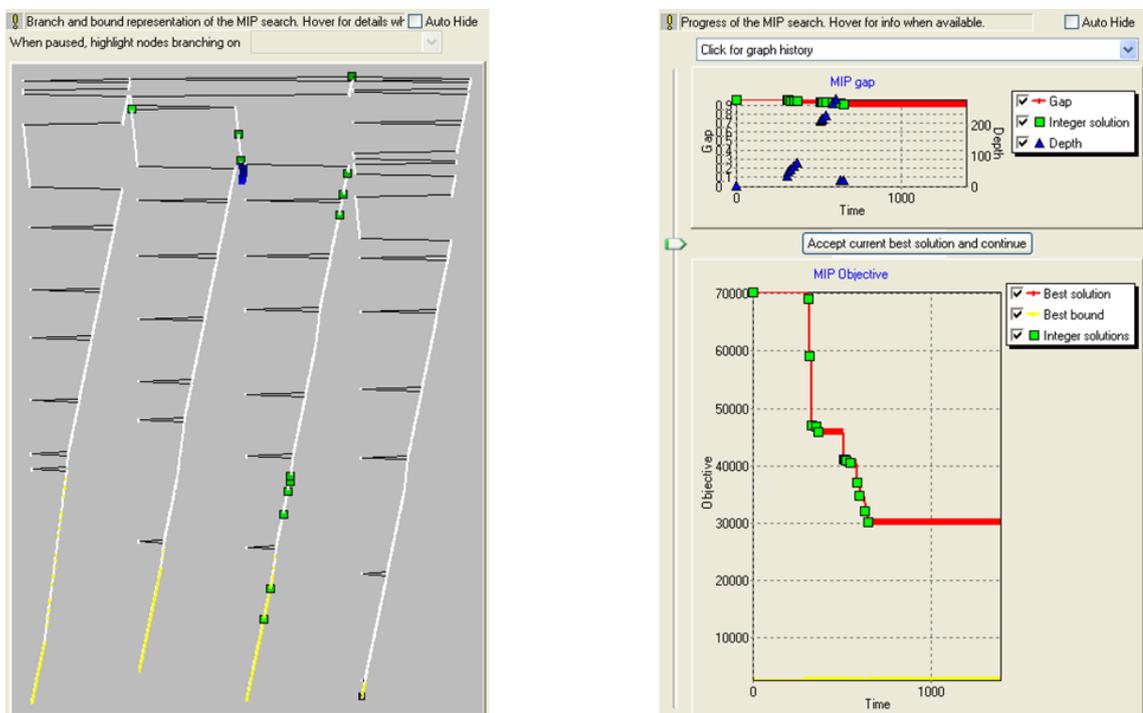


Figure 5.1: Branch and bound and progress of the MIP search

on campus and the pickup scheduling. The statistics according to the mixed linear program optimisation are shown in figure (5.2). It can also be seen there, that the best solution after 24 minutes had an “inconvenience cost” of 30,224.5.

### 5.1 Solutions and Improvement

Currently there are in total 37 bin locations on the campus. Our optimised solution used a total of 40 bin locations of which 24 are not currently used, whereas

Current optimization statistics.			
<b>Matrix:</b>		<b>Presolved:</b>	
Rows(constraints):	10952	Rows(constraints):	7846
Columns(variables):	308560	Columns(variables):	252174
Nonzero elements:	620740	Nonzero elements:	507968
Global entities:	1220	Global entities:	1220
Sets:	0	Sets:	0
Set members:	0	Set members:	0
<b>Overall status: Performing global search...</b>			
<b>LP relaxation:</b>		<b>Global search:</b>	
Algorithm:	Simplex dual	Current node:	2274
Simplex iterations:	5058	Depth:	73
Objective:	2704.88	Active nodes:	1111
Status:	LP Optimal	Best bound:	2704.88
Time:	1.7s	Best solution:	30224.5
		Gap:	91.0507%
		Status:	15 integer solution(s)
		Time:	1447.6s

Figure 5.2: Branch and bound and progress of the MIP search

16 locations match with the current situation. Figure 5.3 gives an overview of a

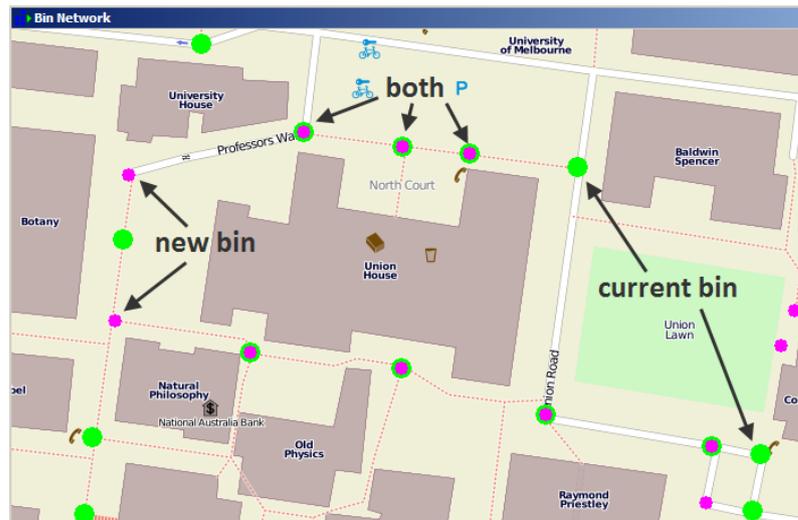


Figure 5.3: New and old bin locations

campus section showing the old and new locations and indicating where they match.

Compared to the results of our model given the current bin placement and using the optimisation we get the improvement in inconvenience cost shown in table (5.1). This shows that we could improve the inconvenience cost by over one third.

The bin pickup scheduling is shown in table (5.2) The detailed routes and the other data can be seen via the graphical user interface of XpressMP as explained in the Appendix.

Beyond that, this GUI makes the data end-user friendly and there is no need for the

Convenient cost <sub>old</sub> :	45,537.5
Convenient cost <sub>new</sub> :	30,224.5
Improvement:	33.63%

Table 5.1: Comparison of convenient cost and improvement

1. route	11:00	campus south-east
2. route	13:00	big tour and campus west
3. route	15:15	mid-campus tour
4. route	16:15	campus east

Table 5.2: Bin pickup scheduling and routes

client to know the source code or to understand the used algorithms completely.

## 6 Conclusion

In this concluding section we discuss a number of challenges we faced, particularly around the reliability of the data, and hence the final results, and provide an indication of other areas the model can be applied to.

As described above, we calculated the model parameters by extrapolating from a single time point sample using a profile captured at a small number of locations for all time periods. This is clearly an approximation and there is no guarantee that the profile at one location matches another. Another factor that introduces uncertainty in our outcomes is that the traffic along edges and rubbish generated at each vertex has a stochastic nature. For example, we have been told by Peter that there is a noticeable increase in the rubbish on warm days compared to cooler days. Furthermore, we only considered time periods of an hour and across a single day. It is quite possible considering that people have different lectures on different days of the week and have lunch at different venues that there is consistency between weeks. After all, lecture time tables are weekly and possibly people have weekly habits for their eating venues. Regardless of how much more data collection we do in future and how much of a stochastic model we introduce, one of the first tasks would be to understand the variability so that we could at least frame our results in some sort of confidence interval.

An activity that can be done without gathering more data is doing sensitivity analysis on the model. By this we mean how much do the results of the optimisation change given changes to the input data. In the previous paragraph, we discussed our concerns about the variability in the traffic and other data. These concerns can be quantified by using such an analysis. A method for implementing this analysis would be to run a loop where random changes are made to the input data and the model rerun. The changes allowed would need to be constrained - for example we would ensure that total traffic was not changed to exceed the number of people on campus or that the total rubbish generated did not exceed the total rubbish collected by the external contractors. This could be run over a long period provided computing resources are available.

We provided only an approximate split between recycling waste and landfill waste based on a simple observation of how full a recycling bin was compared to its neighbouring landfill bin. Further observations would be needed to obtain a more accurate breakdown. Further detail would also be gained if we provided a more detailed breakdown of rubbish type. In this project we considered only coffee cups and disposable

lunch waste. Our data collection also ignored rubbish that might have been brought onto campus, rubbish taken off campus (this would be negligible) and also rubbish that is deposited in bins inside buildings.

With regards to the network model and in particular the edges, we did not exclude from the pickup cart routes the edges that are steps or are unsuitable for the cart.

It is interesting to note that during the course of the project the University has been undertaking a number of initiatives to improve the management of waste. These are to encourage the use of non-disposable coffee containers and the placement of more recycling waste bins (in the form of 'wheelie bins'). Furthermore, it is clear that in society as a whole there is slow change toward better management of waste. This would mean that if this project was run again next year, the results might be different.

Our model has application in other areas where determining a location of some facility and then scheduling visits to that facility is required. For example one other application is vending machine siting. This is the problem of determining sites for vending machines such that the sales from the machine are high and balanced against the cost of visiting the machine to refill it. An interesting extension to this would be to include optimisation of the range of items sold in the machine. Another application area where perhaps scheduling of fill up would require careful managing is with ATM machines that are not attached to the bank premises. For example, ATMs located in convenience stores. These would be sited in locations where demand is high but taking into account high risk areas. We would imagine that refill of ATMs is quite costly given the security requirements and than there might be constraints on when the ATM could be refilled.

A modification to our problem would be that of bin relocating. Given that the campus has bins already in place, we would investigate the problem of relocating existing bins taking into account the cost of doing so. This would include as input existing bins and seek to decrease inconvenience cost whilst minimising relocation cost.

One of the challenges we faced when putting forward the initial project proposal and discussing this with other project sponsors, is that of showing in the proposal value to the client. In most cases, the situation being modeled is one that is already up and running and so the case has to be put forward that the project is worthwhile to undertake to see if an improvement can be made. Clearly, this has to be handled carefully and tactfully. In situations there there are hundreds of thousands of dollars of potential savings and where the project cost is a fraction of this, then the case for the project is clearer. Another approach is that in situations where there is some form of contract governing the activity, as there is in this case, where SERCO are contracted to provide services to Melbourne University, then the model and results might suggest some additions that could be made to the contract to ensure better

service. Alternatively, from the other side, the work could provide indications of where cost savings could be made whilst maintaining the same service levels.

In summary, the challenges we faced were: framing the initial proposal so that we could show value to the client, gathering the data in sensible manner given the time constraints and finally developing a model that is tractable. Finally, whilst our model did show an improvement in convenience cost of 33% we feel that the data we captured, and based the model on, is not reliable enough to provide something that is reasonable to present as a recommendation. As a minimum we should at least frame the result in a confidence interval before doing so.

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- Traffic Flow - The list will show traffic flow in both directions and for all time periods along the edge closest to the mouse cursor. Moving the mouse to another edge will cause the details to be shown for that edge.
- Rubbish Generation - The list will show rubbish generation for all time periods for the node closest to the mouse cursor.
- Routes - The list will show the pickup routes from the optimiser at the times the optimiser schedules pickups for. Clicking on the line will cause the route to be displayed on the map.

Scroll bars on the right hand and bottom edge of the map canvas allow different parts of the map to be viewed. Maximising or otherwise enlarging the window will also allow more of the map to be shown.

The BinNetwork program includes facilities for processing data for input into the optimisation model:

- Reading edge traffic data.
- Reading seating count data.
- Output to a DAT file of rubbish generation for each node for each time period.
- Output to a DAT file of shortest path distances between every node in the network.
- Output to a DAT file of service cost data for each edge for each time period.
- Display of rubbish generation for each node.
- Display of traffic for each edge.
- Display of current bin locations.
- Display of car park and depot locations.
- Display of seating and food outlet locations.

It also includes facilities for reading results from the optimiser and displaying them:

- Input of pickup schedule routes.
- Display of full routes on the map.
- Display of new bin locations on the map.

## 7 Appendix

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09:00	10:00	11:00	12:00	13:00	14:15	15:15	16:15	17:15	$V_{i,max}^p$
People leaving with food - Place 1: Professor's Court Café									
8	11	16	15	14	13	11	5	0	16
People leaving with food - Place 2: Pronto Pizza									
5	35	10	35	55	45	20	10	0	55
People sitting - Place 1: Professor's Court Café									
9	9	8	3	8	17	11	23	13	23
People sitting - Place 2: Union Lawn									
2	11	13	34	40	21	14	11	11	40
People walking - Place 1: Node 56 to 59									
25	66	68	90	63	40	60	53	29	90
People walking - Place 2: Node 1 to 117									
190	120	145	200	205	220	195	200	115	220
People walking - Place 3: Node 124 to 42									
30	75	180	85	340	115	85	75	20	340
People walking - Place 4: Node 42 to 124									
15	115	25	290	40	60	65	105	40	290

Table 7.1: Whole day observations for profiles