

Aufgabe 1

Note Title

7/6/2016

a)

$\tilde{\pi}$	A		
1	2	5	3
2	1	0	5
3	4	6	2

$\rightarrow 2, \rightarrow 1$

$\tilde{\pi}$	A		
3	4	6	2
2	1	0	5
1	2	5	3

π	A		
3	4	6	2
2	$\frac{1}{4}$	$0 - \frac{1}{4} \cdot 6$	$5 - \frac{1}{4} \cdot 2$
1	$\frac{1}{2}$	$5 - \frac{1}{2} \cdot 6$	$3 - \frac{1}{2} \cdot 2$

π	A			
3	4	6	2	\leftarrow
2	$\frac{1}{4}$	$-\frac{6}{4}$	$\frac{9}{2}$	
1	$\frac{1}{2}$	2	2	

π	A			
3	4	6	2	
1	$\frac{1}{2}$	2	2	
2	$\frac{1}{4}$	$-\frac{6}{4}$	$\frac{9}{2}$	

π	A			
3	4	6	2	
1	$\frac{1}{2}$	2	2	
2	$\frac{1}{4}$	$-\frac{3}{4}$	$\frac{9}{2}$	

π	A		
3	4	6	2
1	$\frac{1}{2}$	2	2
2	$\frac{1}{4}$	$-\frac{3}{4}$	6

$$L = \begin{pmatrix} 1 & 0 & 0 \\ \frac{1}{2} & 1 & 0 \\ \frac{1}{4} & -\frac{3}{4} & 1 \end{pmatrix}$$

$$U = \begin{pmatrix} 4 & 6 & 2 \\ & 2 & 2 \\ & & 6 \end{pmatrix}$$

$$P = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$

Π	\mathcal{B}	
1	12	20 20
2	6	8 7
3	3	2 1

Π	\mathcal{B}	
1	12	20 20
2	$\frac{1}{2}$	$8 - \frac{1}{2} \cdot 20$ $7 - \frac{1}{2} \cdot 20$
3	$\frac{1}{4}$	$2 - \frac{1}{4} \cdot 20$ $1 - \frac{1}{4} \cdot 20$

Π	B		
1	r_2	20	20
2	$r_{\frac{1}{2}}$	-2	-3
3	$r_{\frac{1}{4}}$	-3	-4

↷

Π	B		
1	r_2	20	20
3	$r_{\frac{1}{4}}$	-3	-4
2	$r_{\frac{1}{2}}$	-2	-3

↷

Π	B		
1	r_2	20	20
3	$r_{\frac{1}{4}}$	-3	-4
2	$r_{\frac{1}{2}}$	$\frac{2}{3}$	-3

$-3 - \frac{2}{3} \cdot (-4)$

Aufgabe 2

Note Title

7/6/2016

Vandermonde-Matrix

$$\begin{pmatrix} 1 & x_0 \\ 1 & x_1 \\ 1 & x_2 \\ \vdots & \vdots \\ 1 & x_{m-1} \end{pmatrix}$$

$$\underbrace{\begin{pmatrix} 1 & -1 \\ 1 & 0 \\ 1 & 1 \\ 1 & 2 \\ 1 & 3 \end{pmatrix}}_A \underbrace{\begin{pmatrix} t \\ m \\ c \end{pmatrix}}_c = \underbrace{\begin{pmatrix} 0 \\ 0 \\ 1 \\ 1 \\ 2 \end{pmatrix}}_y$$

überbestimmtes GS \Rightarrow Normalequation

$$A^T A c = A^T y$$

$$\begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ -1 & 0 & 1 & 2 & 3 \end{pmatrix} \begin{pmatrix} 1 & -1 \\ 1 & 0 \\ 1 & 1 \\ 1 & 2 \\ 1 & 3 \end{pmatrix} \begin{pmatrix} t \\ m \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ -1 & 0 & 1 & 2 & 3 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 1 \\ 1 \\ 2 \end{pmatrix}$$

$$\begin{pmatrix} 5 & 5 \\ 5 & 15 \end{pmatrix} \begin{pmatrix} t \\ m \end{pmatrix} = \begin{pmatrix} 4 \\ 9 \end{pmatrix}$$

$$\Rightarrow 5 \cdot t + 5 \cdot m = 4 \Rightarrow t = \frac{1}{5}(4 - 5m) = \frac{3}{10}$$

$$5 \cdot t + 15m = 9 \Rightarrow 4 - 5m + 15m = 9$$

$$\Rightarrow 4 + 10m = 9$$

$$\Rightarrow m = \frac{1}{2}$$

$$b) y = a_0 + a_1 x + a_2 x^2$$

$$\begin{pmatrix} 1 & x & x^2 \\ 1 & -1 & 1 \\ 1 & 0 & 0 \\ 1 & 1 & 1 \\ 1 & 2 & 4 \\ 1 & 3 & 9 \end{pmatrix} \begin{pmatrix} a_0 \\ a_1 \\ a_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 1 \\ 1 \\ 2 \end{pmatrix}$$

$$\underbrace{\hspace{10em}}_A$$

=> Normalengleichung

$$A^T A c = A^T y$$

$$\begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ -1 & 0 & 1 & 2 & 3 \\ 1 & 0 & 1 & 4 & 9 \end{pmatrix} \begin{pmatrix} 1 & -1 & 1 \\ 1 & 0 & 0 \\ 1 & 1 & 1 \\ 1 & 2 & 4 \\ 1 & 3 & 9 \end{pmatrix} \begin{pmatrix} a_0 \\ a_1 \\ a_2 \end{pmatrix} =$$

$$= \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ -1 & 0 & 1 & 2 & 3 \\ 1 & 0 & 1 & 4 & 9 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 1 \\ 2 \end{pmatrix}$$

$$\begin{pmatrix} 5 & 5 & 15 \\ 5 & 15 & 35 \\ 15 & 35 & 99 \end{pmatrix} \begin{pmatrix} a_0 \\ a_1 \\ a_2 \end{pmatrix} = \begin{pmatrix} 4 \\ 9 \\ 23 \end{pmatrix}$$

$$a_0 \approx 0,2286 \quad a_1 \approx 0,3571$$

$$a_2 \approx 0,0714$$

c)

$$A^T A c = A^T y$$

$$\Rightarrow c = \underbrace{(A^T A)^{-1}}_{A^+} A^T y$$

A^+ : Pseudoinverse

Aufgabe 3

Note Title

7/6/2016

Pseudoinverse $A^+ = V \Sigma_+^{-1} U^T$

$$A^+ = \frac{1}{2} \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \\ 1 & -1 & 1 & -1 \end{pmatrix} \begin{pmatrix} \frac{1}{3} & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$= \frac{1}{12} \begin{pmatrix} 0 & 1 \\ -1 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} -3 & 2 \\ -3 & 2 \\ 3 & 2 \\ 3 & 2 \end{pmatrix}$$

Aufgabe 4

Note Title

7/6/2016

$$\cdot \begin{pmatrix} x \\ n \end{pmatrix} \quad T = \begin{pmatrix} R & t \\ 0^T & 1 \end{pmatrix} \in \mathbb{R}^{4 \times 4}$$

$\in \mathbb{R}^{3 \times 3}$ $\in \mathbb{R}^{3 \times 1}$

$$\Rightarrow T \begin{pmatrix} x \\ n \end{pmatrix} = \begin{pmatrix} R & t \\ 0^T & 1 \end{pmatrix} \begin{pmatrix} x \\ n \end{pmatrix}$$
$$= \begin{pmatrix} Rx + t \\ n \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} x \\ n \end{pmatrix} = T^{-1} \begin{pmatrix} Rx + t \\ n \end{pmatrix} \quad \leftarrow$$

Wir wissen weiterhin

$$T^{-1} \in \mathbb{R}^{4 \times 4}$$
$$T^{-1} = \begin{pmatrix} A & b \\ 0^T & 1 \end{pmatrix}$$

$\in \mathbb{R}^{3 \times 3}$ $\in \mathbb{R}^{3 \times 1}$

$$\Rightarrow \begin{pmatrix} x \\ 1 \end{pmatrix} = \begin{pmatrix} A & b \\ 0^T & 1 \end{pmatrix} \begin{pmatrix} Rx + t \\ 1 \end{pmatrix}$$

$$\Rightarrow x = A(Rx + t) + b \cdot 1 = \underbrace{AR}_I x + At + b$$

Setze $A = R^T$

$$\Rightarrow x = \underbrace{R^T R}_I x + R^T t + b$$

$$x = x + R^T t + b$$

$$\Rightarrow b = \cancel{x} - \cancel{x} - R^T t = -R^T t$$

Also

$$T^{-1} = \begin{pmatrix} R^T & -R^T t \\ 0^T & 1 \end{pmatrix}$$