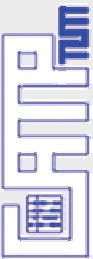


# Linear and Quadratic Subsets for Template-based Tracking



Selim Benhimane<sup>1</sup>, Alexander Ladikos<sup>1</sup>, Vincent Lepetit<sup>2</sup>, Nassir Navab<sup>1</sup>  
<sup>1</sup> Chair for Computer Aided Medical Procedures (CAMP) - TUM, Germany  
<sup>2</sup> Computer Vision Laboratory (CVLAB) - EPFL, Switzerland

## Overview

**Problem**  
What is the best pixel subset to consider during template-based tracking?

**Solution**  
Considering the subset verifying the assumption of linearity of the pixel intensity

with respect to the motion parameters (made during the derivation of the tracking algorithm) improves the tracking performances [Matas et al., ICVGIP 2006].

However, the search for the optimal subset results in a combinatorial explosion when the templates are large.

We propose a simple algorithm that efficiently extracts linear or quadratic subsets in large templates. These subsets are able to cope with high inter-frame motions.

**Results**  
1. Reducing the number of pixels considered improves the efficiency of the tracking algorithm.  
2. The algorithm provides very reliable subsets with less than 30% of the template. It is possible to double the convergence frequency for high inter-frame motions.

**Faster tracking, higher success rate, better performance**

## Extraction Algorithm

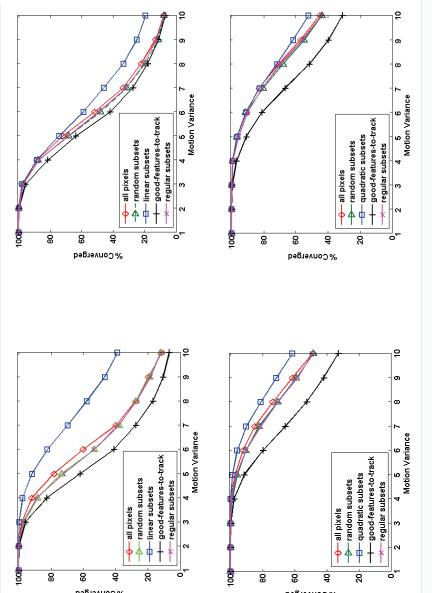
### Basic Ideas

- A linear subset converges in one iteration using the IC algorithm.
- A quadratic subset converges in one iteration using the ESM algorithm.
- Both linear and quadratic subsets are stable under the union operation.

### Proposed Algorithm

```
warp the reference template with N random motions (N > 1)  
set a counter c(1) for each pixel i of the template (initialized to 0)  
for each random motion  $\mathbf{G}_t$   
    for each (3x3) region with center  $\mathbf{P}_i$   
        do one iteration to obtain an estimate  $\mathbf{G}_s$  of  $\mathbf{G}_t$   
        compute  $\mathbf{P} = \mathbf{w}(\mathbf{G}_s)(\mathbf{P}_i)$  // The true new pixel position  
        compute  $\mathbf{P}' = \mathbf{w}(\mathbf{G}_t)(\mathbf{P})$  // The estimated new pixel position  
        if  $\|\mathbf{P} - \mathbf{P}'\| < 1$  // Is one iteration enough to converge?  
            c(i) = c(i) + 1  
        end if  
    end for  
end for  
select the union of the (3x3) regions around the pixels of highest  
counters value
```

## Convergence Frequency



## Linear and Quadratic Subsets

### Template-based Tracking

In an image sequence, find the parameters  $\mathbf{x}$  of the relative motion between the camera and an object using the intensity differences: solve  
$$\operatorname{argmin}_{\mathbf{x}} \| \mathbf{w}(\mathbf{x})(\mathbf{p}) - \mathbf{I}(\mathbf{p}) \|$$

with  $\mathbf{y} = \mathbf{I}(\mathbf{w}(\mathbf{x})(\mathbf{p})) - \mathbf{I}(\mathbf{p})$  where  $\mathbf{I}(\cdot)$  is the current image,  $\mathbf{I}(\cdot)$  is the reference image and  $\mathbf{w}$  is the warping function.

### Inverse Compositional Algorithm (IC)

The algorithm uses a first-order Taylor approximation of  $\mathbf{y}$  with respect to  $\mathbf{x}$ . It solves the problem iteratively at each iteration an incremental warp is estimated:  
$$\mathbf{x} = -(\mathbf{J}_c^T \mathbf{J}_c)^{-1} \mathbf{J}_c^T \mathbf{y}(0)$$

Advantage: Efficient (the Jacobian  $\mathbf{J}_c$  is constant)  
Limitation: Low convergence frequency and rate

### Efficient Second-order Minimization (ESM)

[Benhimane and Mallis, IROS 2004]

The algorithm uses a second-order Taylor approximation of  $\mathbf{y}$  with respect to  $\mathbf{x}$ . Solves the problem iteratively, at each iteration an incremental warp is estimated:  
$$\mathbf{x} = -(\mathbf{J}_{\text{sem}}^T \mathbf{J}_{\text{sem}})^{-1} \mathbf{J}_{\text{sem}}^T \mathbf{y}(0)$$

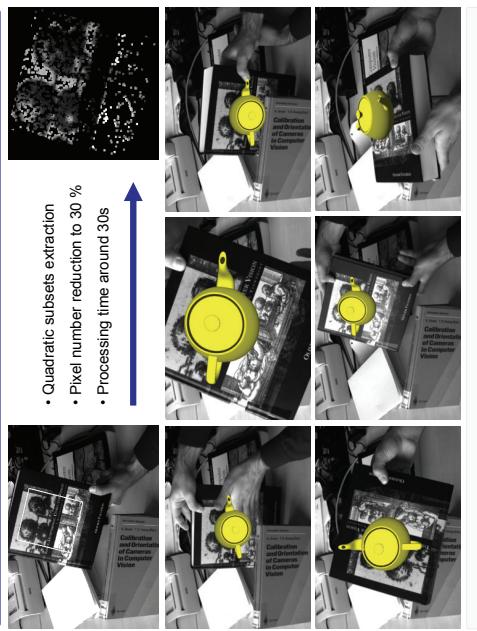
Advantage: Convergence frequency and rate higher than first-order methods  
Limitation: Efficient but less than the IC algorithm ( $\mathbf{J}_{\text{sem}}$  is computed at each iteration)

### Definition of Linear and Quadratic Subsets

A linear subset  $\mathcal{E} = \{\mathbf{p}\}$  verifies: for all motion parameters  $\mathbf{x}$ :  
$$\mathbf{y}(\mathbf{x}) = -\mathbf{J}_{\text{sem}} \mathbf{x}$$

A quadratic subset  $\mathcal{E} = \{\mathbf{p}\}$  verifies: for all motion parameters  $\mathbf{x}$ :  
$$\mathbf{y}(\mathbf{x}) = -\mathbf{J}_{\text{sem}}^T \mathbf{x}$$

## Real-time Template-based Tracking



First row: results obtained with the IC algorithm; second row: results obtained with the ESM algorithm; left column: without noise; right column: in the presence of noise. With the IC algorithm, the linear subsets performs at least two times better than the other methods for large motions. With the ESM algorithm, the differences are smaller, but the quadratic subsets still outperforms all other methods.