

The Minimum Cost Connected Subgraph Problem in Medical Image Analysis

– Supplementary Material –

Markus Rempfler¹, Bjoern Andres², and Bjoern H. Menze¹

¹ Institute for Advanced Study & Department of Informatics,
Technical University of Munich, Germany

² Max Planck Institute for Informatics, Saarbrücken, Germany

A Additional Experimental Results

Details on the *objective-dependent constraints* experiment with 25 subsampled 64×64 problem instances are given in Table 1 and Fig. 1.

For the full datasets, additional experimental results *per instance* for the different constraint generation strategies are shown in Table 2 and Table 3. The gap is calculated as $\text{gap} = 1 - \frac{\text{objective}}{\text{bound}}$ and shown in percent. Note that two strategies might obtain solutions with identical objective value, yet establish different bounds. We limited runtime to maximum 4 h and 8 h for DRIVE and OPF, respectively.

Table 1. Number of solved instances (#S), fastest to solve (#F) and instances with $\text{gap} \leq 1\%$ per strategy.

	without			with		
	#S	#F	# ϵ_1	#S	#F	# ϵ_1
Equidistant	19	2	19	19	4	19
k-Nearest	21	13	21	21	8	21
k-Interleave	19	3	20	21	7	21
Minimal	11	0	11	11	0	15
Interleave	12	2	13	14	1	14
Nearest	10	1	10	11	1	14

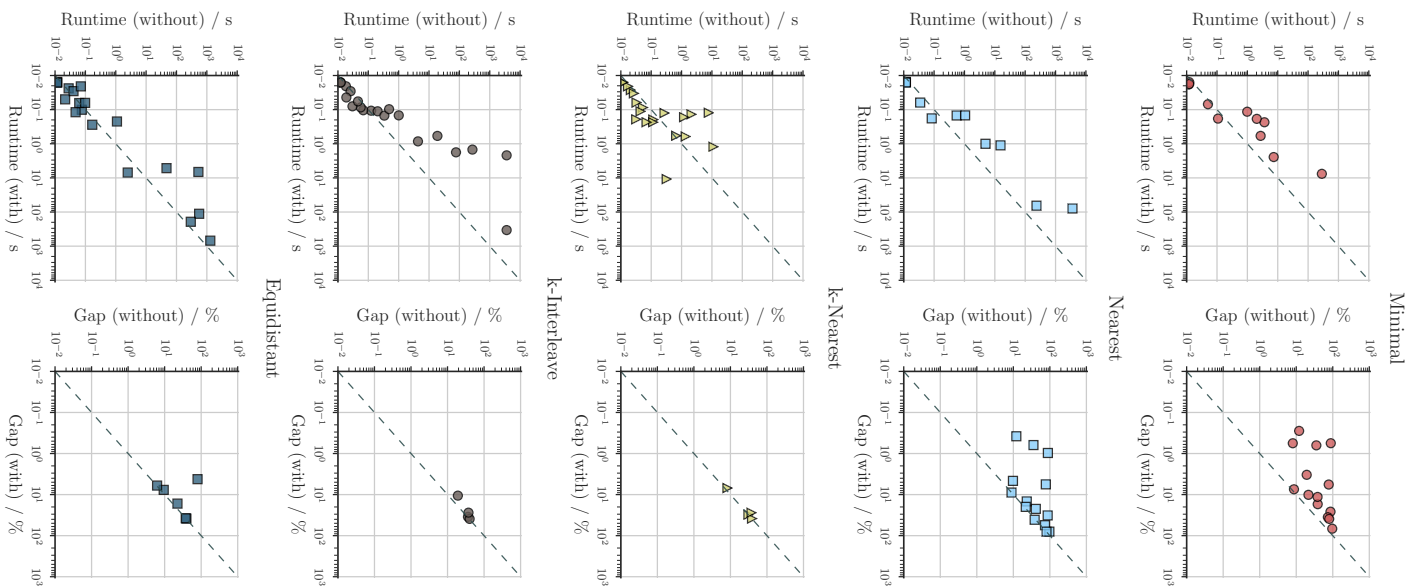


Fig. 1. Comparison of the objective-dependent constraints on the small problem instances. **Left:** Runtime with objective-dependent constraints (abscissae) versus runtime without these additional constraints (ordinate). *Unsolved* instances are compared on the **right:** Gap of established solution-bound pair with the objective-dependent constraints (abscissae) and without (ordinate). We observe a stronger benefit of the additional constraints for instances that take more than a second to be solved. The established gap in unsolved problems tends to be smaller with the additional constraints.

Table 2. Detailed results on the DRIVE dataset. All gaps are shown in %.

	Minimal		Equidistant		Nearest		k-Nearest		k-Interleave		Geodesic		Topocut	
	Objective	Gap	Objective	Gap	Objective	Gap	Objective	Gap	Objective	Gap	Objective	Gap	Objective	Gap
1	-25.221	99.951	-51036.060	0.076	-25.221	99.951	-25.221	99.951	-51034.143	0.009	-51027.773	-	-51033.528	-
2	-6730.676	88.220	-57056.888	0.020	-6730.676	88.220	-57059.422	0.000	-57059.422	0.002	-57059.422	-	-57022.820	-
3	-426.925	98.514	-11566.359	59.671	-426.925	98.515	-10230.143	64.268	-426.925	98.509	-28495.543	-	-28363.606	-
4	-2136.957	95.723	-49915.736	0.009	-2136.957	95.722	-49915.444	0.001	-49915.408	0.006	-49915.736	-	-49917.342	-
5	-52.461	99.883	-44778.050	0.036	-52.461	99.883	-44782.041	0.001	-44782.031	0.008	-44782.190	-	-44743.784	-
6	-2935.626	93.132	-42202.005	1.168	-2935.626	93.132	-42240.223	0.967	-42233.743	0.957	-42620.005	-	-42517.828	-
7	-9227.616	79.193	-44247.003	0.010	-9227.616	79.193	-44246.127	0.005	-44246.892	0.010	-44247.278	-	-44191.763	-
8	-8203.665	76.574	-8204.111	76.543	-8203.665	76.575	-8204.111	76.492	-8203.665	76.481	-34837.718	-	-34784.324	-
9	-44.521	99.853	-30083.291	0.696	-44.521	99.853	-44.522	99.853	-44.521	99.853	-30215.726	-	-30078.909	-
10	-10928.011	74.891	-43360.988	0.220	-10928.011	74.891	-43375.895	0.001	-43375.894	0.002	-43375.380	-	-43340.989	-
11	-218.391	99.548	-48162.391	0.087	-218.391	99.548	-48158.325	0.010	-48157.739	0.022	-48160.034	-	-48129.812	-
12	-2259.923	94.873	-43960.046	0.178	-2259.923	94.873	-43048.430	2.170	-43991.618	0.010	-43991.618	-	-43985.121	-
13	-9378.653	79.817	-46330.246	0.162	-9378.653	79.817	-45934.617	0.879	-46336.008	0.010	-46338.359	-	-46250.965	-
14	-6211.964	86.212	-44790.480	0.435	-6211.964	86.212	-6212.368	86.188	-44952.960	0.021	-44956.168	-	-44919.007	-
15	-43.096	99.901	-43411.817	0.449	-43.096	99.901	-42470.562	2.573	-43554.831	0.030	-43558.363	-	-43531.902	-
16	-9116.216	78.385	-42095.265	0.086	-9116.216	78.386	-9116.693	78.343	-42089.816	0.006	-42090.386	-	-42092.120	-
17	-9824.399	73.731	-37305.838	0.081	-9824.399	73.732	-37302.845	0.008	-37304.001	0.001	-37304.466	-	-37303.848	-
18	-25.801	99.947	-48615.828	0.010	-44154.734	9.243	-48615.519	0.002	-48615.806	0.001	-48615.989	-	-48609.983	-
19	-8593.296	84.006	-52849.344	1.491	-8593.296	84.006	-52717.862	1.541	-8593.296	83.952	-53498.420	-	-53376.441	-
20	-8468.807	79.247	-39938.908	2.005	-8468.807	79.248	-8469.268	79.189	-40146.413	1.347	-40677.896	-	-40648.447	-

Table 3. Detailed results on the OPF dataset. All gaps are shown in %. Topocut has not been available for 3D data.

	Minimal		Equidistant		Nearest		k-Nearest		k-Interleave		Geodesic		Topocut	
	Objective	Gap	Objective	Gap	Objective	Gap	Objective	Gap	Objective	Gap	Objective	Gap	Objective	Gap
1	-28195.261	0.001	-28195.261	0.003	-28198.749	0.003	-28194.760	0.003	-28195.261	0.003	-28195.261	-	-	-
3	-8381.838	28.451	-11709.022	0.009	-8382.906	28.456	-8382.906	28.443	-8381.838	28.431	-11709.022	-	-	-
4	-28337.583	0.014	-28337.583	0.064	-28341.613	0.035	-28337.583	0.000	-28337.583	0.010	-28337.583	-	-	-
5	-3996.556	40.172	-6556.257	1.828	-3997.058	-40.183	-4085.549	38.823	-6671.876	0.008	-6671.876	-	-	-
6	-12892.958	0.674	-12894.707	0.673	-12894.707	0.695	-12908.342	0.008	-12906.592	0.279	-12906.592	-	-	-
7	-11547.678	0.355	-11549.190	0.652	-11549.190	0.438	-11549.190	0.009	-11547.678	0.009	-11547.678	-	-	-
8	-12387.719	6.868	-12399.757	10.182	-12389.380	7.019	-12399.757	2.734	-12400.033	2.781	-12400.033	-	-	-
9	-16675.030	6.032	-17367.460	2.014	-16677.321	6.110	-17032.020	3.026	-17365.075	1.335	-17552.617	-	-	-