Motivation

- Analysis of a group of images requires their alignment
- Alignment of 2D face images, 3D tomosgraphic images, Ultrasound Mosaicing
- Pairwise registration methods with a priori chosen prior lead to bias introduction
- Simultaneous registration presents an interesting alternative but leads to a higher computational complexity because of
  - multivariate similarity measures
  - optimization in higher dimensional parameter space
- Direct estimation of multivariate measures is prohibitive because the dimension grows with the number of images
- We present multivariate similarity metrics and efficient optimization strategies

Multivariate Similarity Measures

- Formulate intensity-based registration as maximum likelihood estimation
  \[ x = \text{arg max} \log p(I_1, I_2, ..., I_n | x) \]
- Approximations to avoid the estimation of the high-dimensional PDF

Efficient Non-Linear Optimization

Transformation Parameterization

Rigid Transformations do not form vector spaces but Lie Groups. We perform a geometric optimization where the group structure is taken care of intrinsically. Special Euclidean Group:

\[ x \in SE(3) \]

The tangent space is the Lie algebra

\[ s(3) = \{ \omega | \omega \in \mathbb{R}^3, \omega^T \omega = -I \} \]

Exponential map: Lie Algebra to Lie Group

\[ \exp: s(3) \rightarrow SE(3) \]

Update step:

\[ y = x \exp(h) = x \exp(n_i h_i) \]

Optimization Methods

Cost function

\[ E(x) = \sum_{i,j} J_{f_i}(x) = \sum_{i,j} \frac{1}{2} |f_{ij}(x)|^2 \]

Taylor series

\[ E(x \exp(h)) \approx E(x) + J_{f_i}(x) \cdot h + \frac{h^T \cdot H_{E}(x) \cdot h}{2} \]

Gauß-Newton

Linear approximation of \( F(x \exp(h)) \)

leading to

\[ F(x \exp(h)) = \sum_{i,j} \frac{1}{2} |f_{ij}(x \exp(h))|^2 \]

\[ \sum_{i,j} \frac{1}{2} f_{ij}(x \exp(h)) f_{ij}(x \exp(h)) \]

Gauss-Newton:

\[ J_{f_i}(x \exp(h)) \approx J_{f_i}(x) + H_{f_i}(x) \cdot h \]

\[ J_{f_i}^{\text{ESM}} = \frac{1}{2} J_{f_i}(x) + J_{f_i}(x \exp(h)) \]

Gradient Calculation

\[ J_{f_i}(x \exp(h)) \approx J_{f_i}(x) + H_{f_i}(x) \cdot h \]

\[ J_{f_i}^{\text{ESM}} = \frac{1}{2} J_{f_i}(x) + J_{f_i}(x \exp(h)) \]

Conclusion

- Strict mathematical deduction of the new similarity class APE
- Relation of APE to the Congealing framework and extension to CMP
- Derivation of steepest descent, Gauß-Newton, and ESM for APE
- Application of ESM to intrinsically non-squared similarity metrics CC, CR, MI
- Evaluation on 3D ultrasound data

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