

# Construction of Statistical Shape Models from Minimal Deformations

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**Abstract.** Statistical shape models (SSM) capture the variation of shape across a population, in order to allow further analysis. Previous work demonstrates that deformation fields contain global transformation components, even if global pre-registration is performed. It is crucial to construction of SSMs to remove these global transformation components from the local deformations - thus obtaining *minimal deformations* - prior to using these as input for SSM construction. In medical image processing, parameterized SSMs based on control points of free-form deformations (FFD) are a popular choice, since they offer several advantages compared to SSMs based on dense deformation fields. In this work, we extend the previous approach by presenting a framework for construction of both, unparameterized and FFD-based SSMs from minimal deformations. The core of the method is computation of minimal deformations by extraction of the linear part from the original dense deformations. For FFD-based SSMs, the FFD-parameterization of the minimal deformations is performed by projection onto the space of FFDs. Both steps are computed by close-form solutions optimally in the least-square sense. The proposed method is evaluated on a data set of 62 MR images of the corpus callosum. The results show a significant improvement achieved by the proposed method for SSMs built on dense fields, as well as on FFD-based SSMs.

## 1 Introduction

In this paper we consider the construction of statistical shape models (SSM) from deformation fields which are computed by a non-linear registration method.<sup>4</sup> This approach is discussed for example in [1] and [2], while the advantages of this approach for medical applications are discussed in [1].

Previous work [3, 4] demonstrates that in general, computed deformations contain a substantial amount of global linear transformation, even if global pre-registration is performed, please compare Fig. 1. In [4], an approach is presented to obtain *minimal deformations* by extracting the global similarity transformation from dense deformation fields in a post-processing step. Since SSMs should only describe variations in shape

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<sup>4</sup> Such shape models are also referred to as statistical deformation models (SDM).

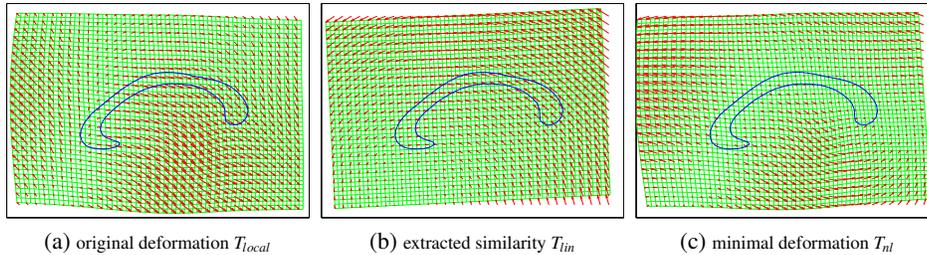


Fig. 1: Illustration of the discussed setting. On a pair of pre-registered images of corpus callosum, nonlinear registration is performed, resulting in a deformation  $T_{local}$  (a). A similarity component  $T_{sim}$  (b) is extracted from the original field  $T_{local}$ , resulting in a minimal deformation  $T_{min}$  (c).

and not e.g. in position, it is intuitive that they should be constructed on minimal deformations, rather than on deformations which still contain global linear transformation components such as similarity. It is shown in [4] that it is not a minor effect, but crucial to build SSMs on minimal deformation fields, in order to obtain correct shape models. If similarity transformation components are not removed from deformation fields prior to SSM construction, this will in general lead to shape models in which the first modes do not necessarily describe the largest variations in shape [4], see also Fig. 3.

The major contribution of this work is to extend the previous approach to a framework, in which we can create SSMs based on minimal deformations not only on dense deformation fields, but also for deformations modeled as FFDs.<sup>5</sup> This extension allows groups using FFD-based SSMs to easily incorporate the advantages of using minimal deformations for SSM construction, thus improving their results. FFD-based SSMs are a popular choice due to the wide usage of FFDs for modeling displacement fields for deformable registration [5, 6] and further advantages, such as lower memory consumption or the possibility to construct models capturing shape variations on different resolution levels [1, 2]. Thus we consider the proposed method to be an important contribution.

## 2 Methods

The backbone of the method is the computation of minimal deformations from original dense deformations (B) - please compare Fig. 2 for the following. This step is discussed in Sec. 2.1. SSMs based on dense deformations can be now constructed from minimal deformations (C), as described in Sec. 2.4.

For deformations modeled by FFDs, the following extension is proposed. In a first step, the deformation is represented on the dense level (A). Then, after computation of the minimal version of the deformation on the dense level, the resulting minimal deformation is transferred to FFD representation (D). From here, the FFD-based SSM can be constructed (E), compare Sec. 2.4.

The major requisite for the proposed framework is the ability to switch between the dense and FFD representation of deformations. While the step from FFD to dense

<sup>5</sup> For brevity, we refer to SSMs built on FFD parameters as *FFD-based SSMs*.

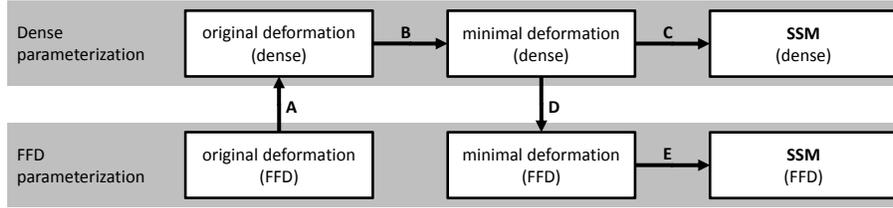


Fig. 2: Sketch of the proposed framework

representation (A) is exact and simply performed by evaluation of the FFD function with given parameters [5], the transfer of dense fields onto FFD models (D) involves a projection and presents an approximation for general fields. This step is presented in Sec. 2.3.

In the following, we present the single methods of the proposed framework.

## 2.1 Computing Minimal Deformations

In the context of registration, the transformation which aligns the target and source images  $I_T$  and  $I_S$  is a function  $T : \Omega \rightarrow \Omega$  where  $\Omega \subset \mathbb{R}^d$  is the image domain of dimension  $d = 2, 3$ .  $T$  is assumed to establish point-to-point correspondence after the registration, such that  $I_T$  and  $I_S \circ T$  are aligned. In most of the current methods for deformable scenarios, the transformation  $T$  is composed of a global, linear transformation  $T_{global}$  and the non-linear local part  $T_{local}$ , resulting in  $T = T_{global} \circ T_{local}$ . In general, the global part is computed prior to the local component and no joint computation of the two terms is employed.

We model non-linear transformations as an addition of the identity function  $\text{Id}$  and a certain displacement field  $U$ , and get  $T_{local} = \text{Id} + U$  with  $T_{local}(X) = Y$ .

Our goal is to extract the remaining linear transformation component from a given deformation. To this end, we model the deformation as a composition of a linear part  $T_{lin}$  and a non-linear part  $T_{nl}$ , that is  $T_{local} = T_{lin} \circ T_{nl}$ .

The task now is to estimate  $T_{lin}$  and  $T_{nl}$ , such that  $T_{nl}$  becomes minimal in some meaningful sense. We reformulate  $T_{local} = T_{lin} \circ T_{nl}$ , to arrive at a form which allows us to minimize the norm of the displacement field  $U_{nl}$  of the non-linear component  $T_{nl}$  with respect to the linear transformation  $T_{lin}$ .

$$T_{local}(X) = (T_{lin} \circ T_{nl})(X) \quad (1)$$

$$Y = T_{lin}(X + U_{nl}(X)) \quad (2)$$

$$T_{lin}^{-1}(Y) - X = U_{nl}(X) . \quad (3)$$

Please note that in step from (2) to (3), the invertibility of  $T_{lin}$  is assumed.

Thus, we can define a cost function, the optimization of which results in a linear transformation (described by parameters  $p$ ), such that the norm of the vectors of the displacement field becomes minimal with respect to the mean squared norm. The cost function  $E$  for dense displacement fields discretized by  $n$  points is given by

$$E(X, Y, p) = \frac{1}{n} \sum_i \|U_{nl}(X_i)\|^2 = \frac{1}{n} \sum_i \|X_i - T_{lin}^{-1}(Y_i; p)\|^2 , \quad (4)$$

where the respective minimization is  $p = \arg \min_{p'} E(X, Y, p')$ .

Once the linear transformation  $T_{lin}^{-1}$  is computed, the corresponding displacement field is given by Equation (3) and the minimal deformation is  $T_{nl} = \text{Id} + U_{nl}$ .

Please note that the minimal deformation  $T_{nl}$  is expressed in the reference frame of the target image, which is important for application to SSMs, compare [2].

The above derivation is valid for any invertible transformation  $T_{lin}$ . Since the optimization problem is defined on corresponding points, we can use any transformation type, for which there is a method for point-based registration with given correspondences. In Sec. 2.1.1 we present a collection of implemented methods which go beyond [4]. For space reasons we give references to literature rather than detailed descriptions.

### 2.1.1 Supported Linear Transformation Types

In the presented framework, computation of minimal deformations with respect to rigid, similarity and affine transformations is implemented. All methods are based on techniques for solving the problem of registration of two point sets based on given correspondences, have a closed-form solution and are optimal in the least-squares sense. The similarity transformation is computed by the Umeyama method [7] for the absolute orientation problem, and was used in [4]. The estimation of the rigid transformation can be seen as a modification of the Umeyama method by fixing the scale parameter, and is discussed for example in [8]. The method for the affine transformation can be seen as a constrained version of the direct linear transform (DLT) method, compare for example [9]. Further references can be found in for example in [10].

## 2.2 Free-form Deformations

In this work we focus on the B-Spline FFD to model deformations [5]. However, since our method only uses the assumption of FFDs being a linear combination of certain basis functions, the presented method can easily be adapted to other FFD models. In this general form an FFD-based deformation can be written as  $T(x) = x + U(x)$ , with  $U(x) = \sum_{k=1}^K B_k(x)c_k$ , where  $B_k$  are the basis functions, weighted by  $K$  control points  $c_k$  on a uniform grid. The values  $c_k$  are the parameters of the FFD-model. For the case of B-Spline basis functions, the displacement of a 2D point  $x$  can be expressed by a combination of univariate cubic B-Spline functions as

$$U(x) = \sum_{l=0}^3 \sum_{m=0}^3 B_l(u)B_m(v)c_{i+l,j+m} \quad (5)$$

where  $i = \lfloor x/\delta_x \rfloor - 1$ ,  $j = \lfloor y/\delta_y \rfloor - 1$ ,  $u = x/\delta_x - \lfloor x/\delta_x \rfloor$ , and  $v = y/\delta_y - \lfloor y/\delta_y \rfloor$  where  $B_l$  represents the  $l$ th basis function of the B-Spline, and  $\delta_x = \frac{n_x}{L_x - 1}$  and  $\delta_y = \frac{n_y}{L_y - 1}$  denote the control point spacing for an  $L_x \times L_y$  FFD grid and a discrete image of size  $n_x \times n_y$ . For details and the extension to 3D, please refer to [5].

## 2.3 Projection of Dense Displacement Fields onto FFD Models

A crucial step of the proposed method is the ability to project dense deformation fields onto the used FFD model. This is done by computing the parameters of the FFD model,

such that the difference between the dense minimal deformation field, and the approximation by the FFD parameterization is minimal in the least-squares sense. To this end, a closed-form solution is available [8, 11].

As defined above, a dense displacement field  $U$  is modeled by an FFD transformation as  $y = U(x) = \sum_{k=1}^K B_k(x)c_k$ . For a set of points  $X = [x_1 \dots x_n]$  and  $Y = [y_1 \dots y_n]$ ,  $U$  can be written as  $Y = U(X) = CB(X)^\top$ . With some abuse of notation we have

$$B(X) = [B(x_1) \dots B(x_n)] = \begin{bmatrix} B_1(x_1) & \dots & B_K(x_1) \\ \vdots & & \vdots \\ B_1(x_n) & \dots & B_K(x_n) \end{bmatrix} \in \mathbb{R}^{n \times K}, \quad (6)$$

and  $C = [c_1 \dots c_K]$ . In our case we have  $c_i, y_i, x_i \in \mathbb{R}^d$ .

The least-squares solution for  $C$  can be given as  $C = YB(X)(B(X)^\top B(X))^{-1}$ . The sparse structure of the matrix  $B(X)$  allows the computation also for large 3D problems. Details can be found in [11].

## 2.4 Construction of SSMs

Principal component analysis (PCA) is the preferred method for statistical shape models [1, 2]. The attractive properties of the PCA for shape modeling include optimal linear reconstruction of the data set variance, the estimated modes of variation are orthogonal and uncorrelated, and a closed form solution exists for calculating the principal components at a relatively low computational cost.

The shape model is built from  $m$  column vectors  $v_i$ , also written as a matrix  $V = [v_1 \dots v_m]$ . The vectors represent either linearized displacement fields, or control point sets representing displacement fields, depending on the SSM type.

Given  $V$ , a linear shape model which approximates a given vector  $v$  is given by  $\bar{v}$  and  $\Phi$  as  $v = \bar{v} + \Phi b$ . Here  $\bar{v}$  is the mean  $\bar{v} = \frac{1}{m} \sum_{i=1}^m v_i$ . The matrix  $\Phi$  is constructed from the first  $k$  eigenvectors  $\Phi_i$  of the covariance matrix  $C = \frac{1}{m-1} \bar{V} \bar{V}^\top$ , where  $\bar{V}$  is the de-meaned version of  $V$ . The eigenvalues corresponding to  $\Phi_i$  are denoted by  $\lambda_i$ . The eigenvectors  $\Phi_i$  are also referred to as *modes*. Finally,  $b \in \mathbb{R}^k$  is the parameter vector, describing the scaling of the principal modes needed to approximate  $v$  by the model.

With respect to the proposed framework, the only modification of the standard model construction process is that instead of the original displacement fields or control point sets, we use the respective representation of the corresponding minimal deformation fields, from which the maximum amount of the respective linear transformation is extracted by our method.

## 3 Results and Evaluation

In this section, we demonstrate the improvements achieved by the proposed method and evaluate the critical steps. The improvement of SSMs by our method is discussed in Sec. 3.2. The only component of the framework which includes an approximation is the projection of minimal deformations onto the FFD model. In Sec. 3.3 we show that this error is negligible and has no negative impact on the resulting FFD-based SSMs.

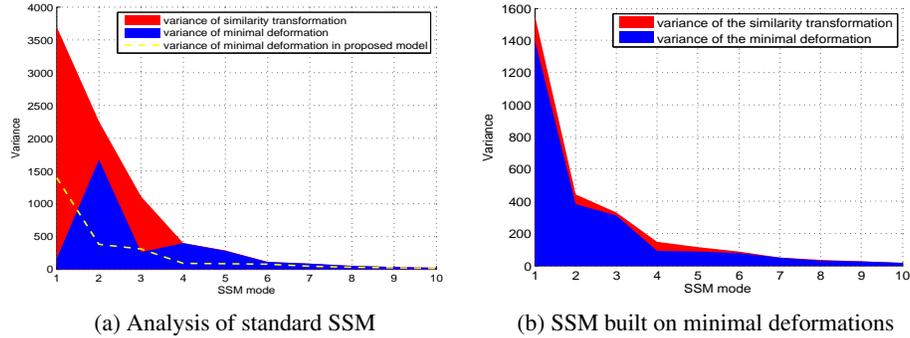


Fig. 3: Analysis of single modes of SSMs built on corpus callosum data. We evaluate the contribution of the similarity transformation to the variance of each mode. **(a)** illustrates an SSM constructed from the original deformations, for which several modes capture more information about similarity transformation than shape. For comparison, the variance of the FFD-based SSM built from minimal deformations ( $SSM_{\text{MIN,FFD}}$ ) is superimposed. In **(b)**, we observe a significant improvement for  $SSM_{\text{MIN,FFD}}$ . The remaining similarity is due to approximation made by projection of minimal deformations and the approximate orthogonality of similarity and minimal deformation components.

### 3.1 The Setting: Test Data, Global and Local Registration

We test the proposed methods on real data which is part of the LADIS (Leukoaraiosis And DISability) study [12]. The data set consists of 62 2D MR images of the midsagittal cross-section of the corpus callosum. Each image is equipped with 72 landmarks selected by clinical experts where the correspondences between the images are known. For an example image of the data set, please see Fig. 1.

The global pre-registration is performed by a similarity transformation estimated by minimizing the squared norm on the landmark correspondences.

In order to obtain the dense deformation fields, we use a recently proposed FFD-based non-linear registration method using discrete optimization [13]. The method computes the deformations  $T_{local_i}$  by registering the images  $I_i$  to the average image of the data collection. After a whole run of 62 registrations, the average is recomputed to obtain an improved version. This is done until the average does not change anymore. It can be shown that under an assumption of Gaussian distribution of the noise, this choice of reference image is optimal with respect to achieving an unbiased coordinate frame for the shape model [14].

The following evaluations are performed with respect to similarity transformations.

### 3.2 Evaluation of Resulting Shape Models

Using non-minimal deformations for SSM construction in general leads to linear transformation components in the resulting SSM, and can lead to SSMs in which the first modes do not describe the largest variations in shape [4].

In order to evaluate this negative influence for a given SSM, we decompose the single modes into a linear and a minimal deformation part as proposed by our method. We measure the influence of the linear part for a given mode by the explained variance

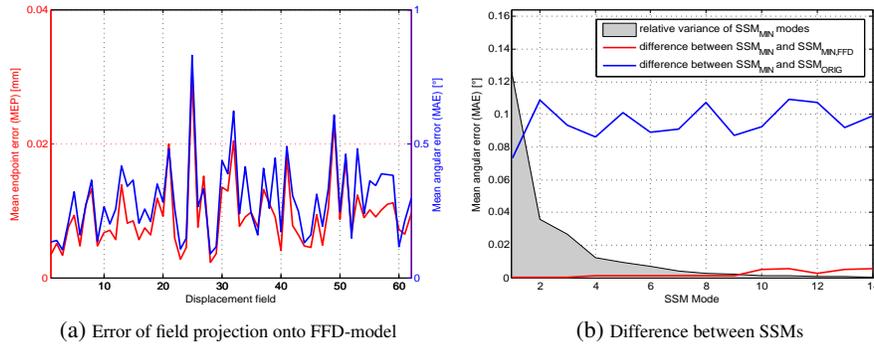


Fig. 4: **(a)** Approximation error caused by projection of the 62 minimal deformations onto a  $9 \times 12$  FFD model. The low error values of less than  $0.03\text{mm}$  and  $0.8^\circ$  demonstrate the quality of the projection onto the FFD model. **(b)** Comparison of modes SSM based on dense minimal deformations ( $\text{SSM}_{\text{MIN}}$ ) and the FFD-based SSM ( $\text{SSM}_{\text{MIN,FFD}}$ ). The relation to the difference between the modes of  $\text{SSM}_{\text{MIN}}$  and the SSM based on original dense fields ( $\text{SSM}_{\text{ORIG}}$ ) shows that the error introduced to  $\text{SSM}_{\text{MIN,FFD}}$  is negligible for the relevant modes with large variance.

corresponding to the linear component. This is determined by distributing the variance of the original mode to the linear and minimal deformation components proportional to the squared norms of the components. This is possible, since the decomposition of the original deformation into a linear and a minimal deformation component can be seen as orthogonal in very good approximation [4]. The results of this experiment are summarized in Fig. 3 and clearly demonstrate the improvement achieved by the proposed method.

### 3.3 Influence of Projection of Fields onto the FFD Model on Resulting FFD-based Shape Models

The following evaluation is performed in order to assure that the projection of the minimal deformations onto a given FFD model does not introduce errors which affect the generation of the FFD-based SSM.

To this end, we measure the approximation error made by the projection of minimal fields. We compute the difference between the minimal fields and their projections by evaluating the standard measures for comparison of displacement fields, the Mean Angular Error (MAE) and Mean Endpoint Error (MEP) [15]. Results in Fig. 4a show that the error is minimal.

To assure that this small error does not influence the final SSM, we compute the difference between the FFD-based SSM and the SSM based on minimal deformations ( $\text{SSM}_{\text{MIN,FFD}}$ ) by evaluating the MAE. To show the impact of this difference, which is caused by the approximation error, we also compute the MAE of the difference between the SSM built on original deformations and  $\text{SSM}_{\text{MIN,FFD}}$ , which might be interpreted as the improvement of the SSM achieved by using minimal deformations. Relating these two differences shows that the approximation error is insignificant when compared to the improvement for the modes with a large variance, please compare Fig. 4b.

## 4 Discussion and Conclusion

The basic ideas underlying the proposed methods are the interpretation of displacement fields as corresponding point sets and the ability to switch between representations of deformations.

In summary, in this paper we propose a framework for construction of SSMs based on minimal deformations. The proposed method is shown to significantly improve the quality of the resulting SSM. The construction of FFD-based SSMs based on minimal deformations is introduced, extending the previous approach. Finally, we extend the previous work by allowing the computation of minimal deformations with respect to rigid and affine transformations.

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