

Exercises in 3D Computer Vision

The submission deadline for this exercises will be June 14, 2005 (5:00pm).

Exercise 22 (P) Epipolar Geometry, Epipolar Lines, and Epipoles

The epipolar geometry is the intrinsic projective geometry between two views and it depends only on the cameras' intrinsic parameters and relative pose (rotation and translation between the camera centers). The fundamental matrix is a 3×3 matrix with rank 2. A 3D point in space is imaged on the first view as \mathbf{x} and in the second view as \mathbf{x}' (see figure 1). The image points satisfy the

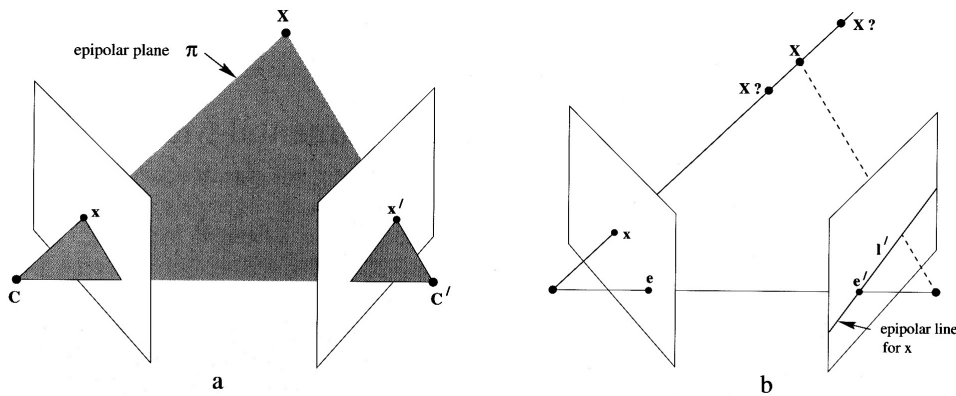


Fig. 9.1. **Point correspondence geometry.** (a) The two cameras are indicated by their centres C and C' and image planes. The camera centres, 3-space point X , and its images \mathbf{x} and \mathbf{x}' lie in a common plane π . (b) An image point \mathbf{x} back-projects to a ray in 3-space defined by the first camera centre, C , and \mathbf{x} . This ray is imaged as a line l' in the second view. The 3-space point X which projects to \mathbf{x} must lie on this ray, so the image of X in the second view must lie on l' .

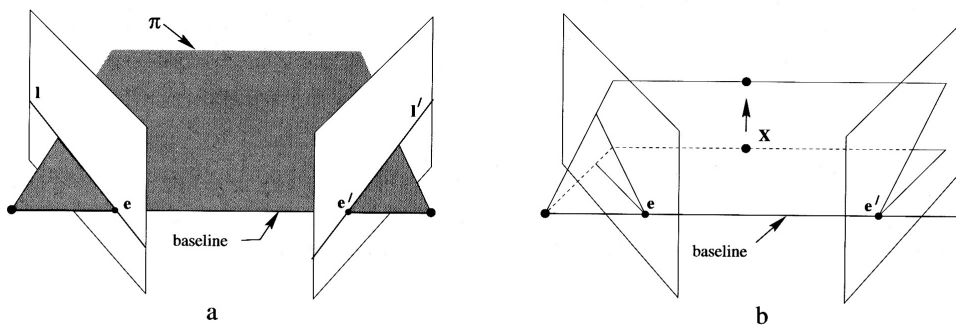


Fig. 9.2. **Epipolar geometry.** (a) The camera baseline intersects each image plane at the epipoles e and e' . Any plane π containing the baseline is an epipolar plane, and intersects the image planes in corresponding epipolar lines l and l' . (b) As the position of the 3D point X varies, the epipolar planes "rotate" about the baseline. This family of planes is known as an epipolar pencil. All epipolar lines intersect at the epipole.

Figure 1: From [HZ03] (p. 240).

equation $\mathbf{x}'^T \mathbf{F} \mathbf{x} = 0$, where \mathbf{F} is the fundamental matrix. Detailed information about the epipolar geometry and fundamental matrix can be found in Hartley & Zissermann [HZ03] (Chapter 9-11).

Given a pair of images (`mag_table_0.jpg` and `mag_table_1.jpg`) and the fundamental matrix \mathbf{F} (`Fmat.mat`) you will have to compute the corresponding epipolar line for all selected image points.

Write an m-file `solution07e22.m` that loads the two images. Use the provided fundamental matrix \mathbf{F} to compute the corresponding epipolar lines. Display the epipolar lines in both images and validate visually that the lines go through the selected points in the other image.

- Use `ginput` to select arbitrary image points.
- If $\mathbf{l}' = \mathbf{F}\mathbf{x}$ where \mathbf{x} is the point in the first view and \mathbf{l}' its corresponding line in the second view, then $\mathbf{l} = \mathbf{F}^T \mathbf{x}'$ computes the epipolar lines in the other image using the same fundamental matrix.
- Use a function `hline.m` to visualize the lines (provided on the webpage of last year's 3D Computer Vision class).

Exercise 23 (P) Basic Estimation of the Fundamental Matrix

Given a set of corresponding points it is possible to estimate the fundamental matrix. In this exercise we will apply a very basic estimate of the fundamental matrix.

The fundamental matrix is defined by the equation $\mathbf{x}'^T \mathbf{F} \mathbf{x} = 0$ for any pair of matching points in two views (this means that \mathbf{x} and \mathbf{x}' correspond to the same 3D point in space).

Solving this equation provides us the equation $x'x f_{11} + x'y f_{12} + x'f_{13} + y'x f_{21} + y'y f_{22} + y'f_{23} + x f_{31} + y f_{32} + f_{33} = 0$. For each set of matching points we get one equation for our linear equation system

$$\mathbf{A}\mathbf{f} = \begin{bmatrix} x'_1 x_1 & x'_1 y_1 & x'_1 & y'_1 x_1 & y'_1 y_1 & y'_1 & x_1 & y_1 & 1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ x'_n x_n & x'_n y_n & x'_n & y'_n x_n & y'_n y_n & y'_n & x_n & y_n & 1 \end{bmatrix} \mathbf{f} = 0.$$

Using the singular value decomposition we can solve the 9-vector \mathbf{f} made up by the entries of \mathbf{F}' .

In order to get the best results for the solution of the linear equation system normalize the points before you build the matrix \mathbf{A} . Normalization means to transform all points such that the centroid of the points is $(0, 0, 1)^T$ and the mean distance to the centroid is $\sqrt{2}$.

After the fundamental matrix of the normalized points is computed, denormalize the fundamental matrix by applying $\mathbf{F} = \mathbf{T}'_2 \mathbf{F}' \mathbf{T}_1$, where \mathbf{T}_1 is the transformation to normalize the points in the first image, \mathbf{T}'_2 is the transformation to normalize the points in the second image, \mathbf{F}' is the fundamental matrix computed with the normalized points, and \mathbf{F} is the fundamental matrix for the original points.

Compute the fundamental matrix \mathbf{F} with the method described above. Load the images `mag1.jpg` and `mag2.jpg`. Select at least eight points in the same order in both images. Use *exercise 22* to compute the epipolar lines of the other view and visualize them. What do you observe for the epipole? What are the properties of the fundamental matrix \mathbf{F} in terms of its determinant and its rank?

Exercise 24 (P) Estimation of the Fundamental Matrix Enforcing the Singularity Constraint

As seen in the previous exercise the estimation of \mathbf{F} without any constraints will give epipolar lines ($\mathbf{l}' = \mathbf{F}\mathbf{x}$) that do not meet in one common epipole.

Use the estimate of \mathbf{F} from the previous exercise as an initial estimate. Enforce now the constraint for singularity by replacing \mathbf{F} with \mathbf{F}' , which is the closest singular matrix to \mathbf{F} under the Frobenius

$$\text{norm } \|\mathbf{F} - \mathbf{F}'\|_F = \sum_{i=1}^3 \sum_{j=1}^3 (f_{ij} - f'_{i,j})^2.$$

Hint: Use the svd on \mathbf{F} . Enforce the smallest value of the diagonal matrix to be zero and thus build \mathbf{F}' which is the closest singular matrix to \mathbf{F} under the Frobenius norm.

References

- [HZ03] Richard Hartley and Andrew Zisserman. *Multiple View Geometry in Computer Vision*. Cambridge University Press, 2nd edition, 2003.