

3D Computer Vision Camera Models

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History :: Camara Obscura

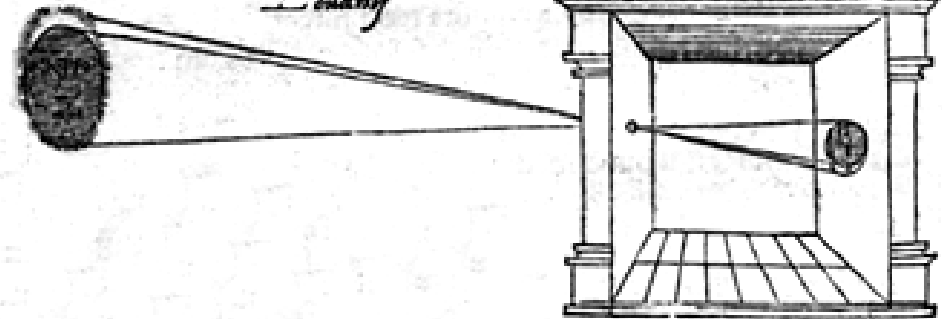
Camera obscura ([|Camera obscura] [LL. camera chamber + L. obscurus, obscura, dark.] (Opt.)

1. An apparatus in which the images of external objects, formed by a convex lens or a concave mirror, are thrown on a paper or other white surface placed in the focus of the lens or mirror within a darkened chamber, or box, so that the outlines may be traced.
2. (Photog.) An apparatus in which the image of an external object or objects is, by means of lenses, thrown upon a sensitized plate or surface placed at the back of an extensible darkened box or chamber variously modified; - commonly called simply the camera.

Websters Dictionary, 1913

illum in tabula per radios Solis, quam in cœlo contingit: hoc est, si in cœlo superior pars deliquiū patiatur, in radiis apparebit inferior deficere, vt ratio exigit optica.

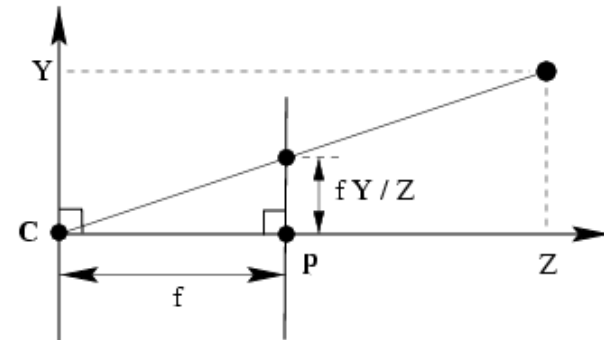
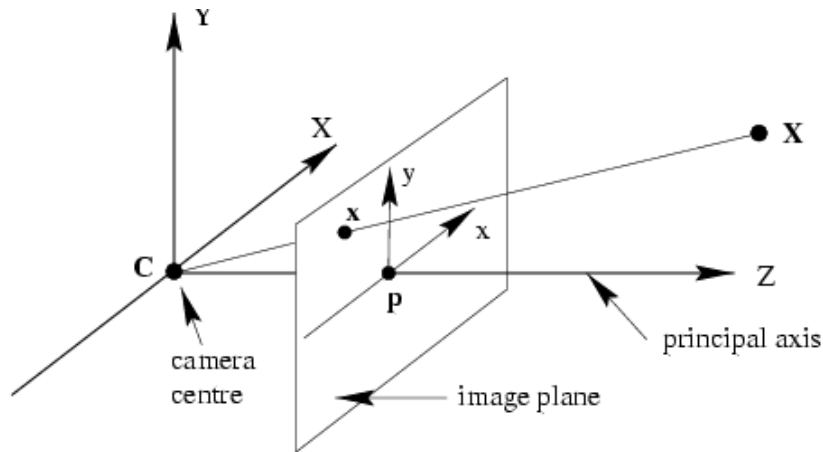
*Solis deliquium Anno Christi
1544. Die 24. Januarij
Louanij*



Sic nos exactè Anno .1544. Louanii eclipsim Solis obseruauimus, inuenimusq; deficere paulò plus q̄ dex-

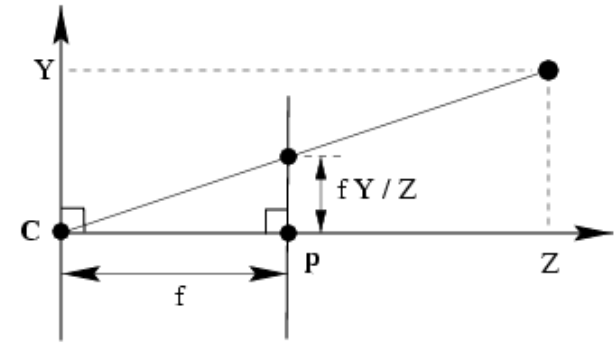
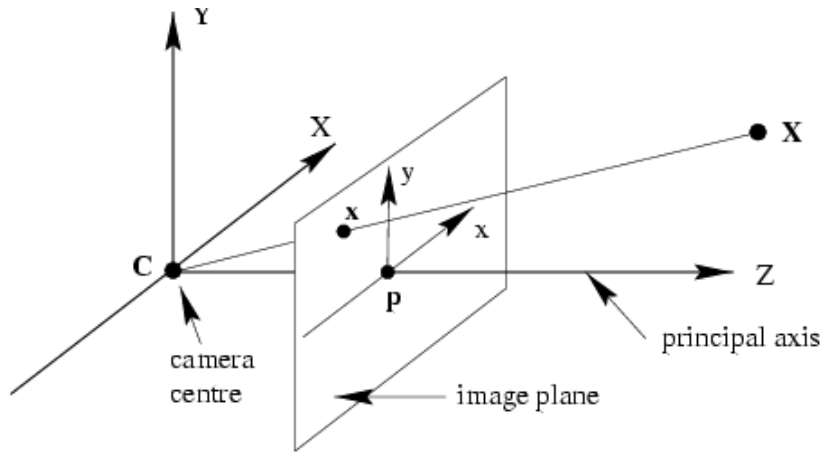
Camera Obscura, Reinerus Gemma-Frisius, 1544, published in "De Radio Astronomica et Geometrica", 1545 from Gernsheim, H., "The Origins of Photography"

Pinhole Camera Model



- mapping between 3D world and 2D image $(X, Y, Z)^T \mapsto (x, y)^T$
- central projection
- models are described in matrices with particular properties

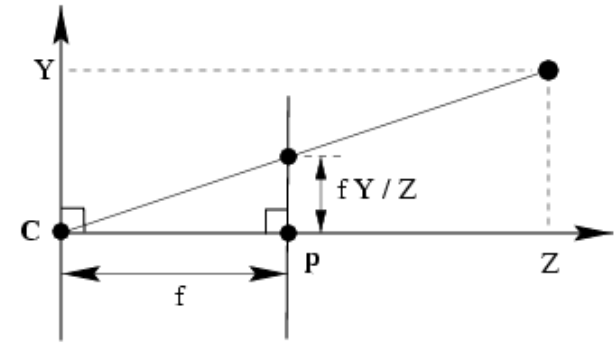
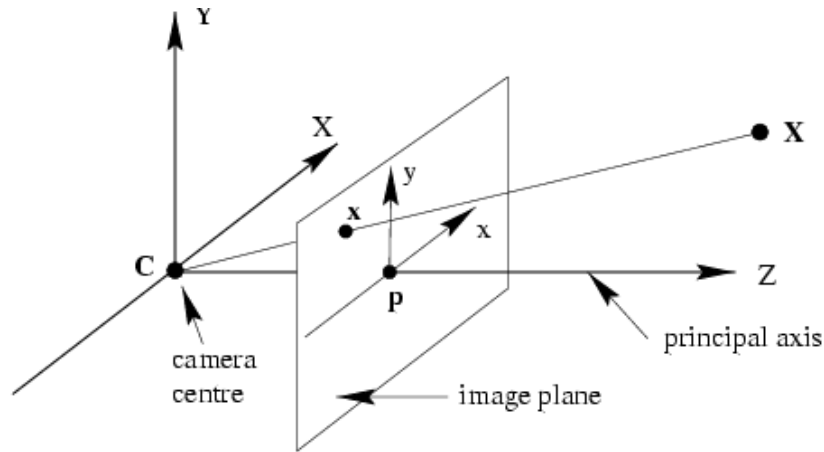
Homogeneous Coordinates



$$(X, Y, Z, 1)^T \mapsto (x, y, z)^T$$

$$\begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix} \mapsto \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} fX \\ fY \\ Z \end{pmatrix} = \begin{bmatrix} f & & & 0 \\ & f & & 0 \\ & & 1 & 0 \end{bmatrix} \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix}$$

Central Projection

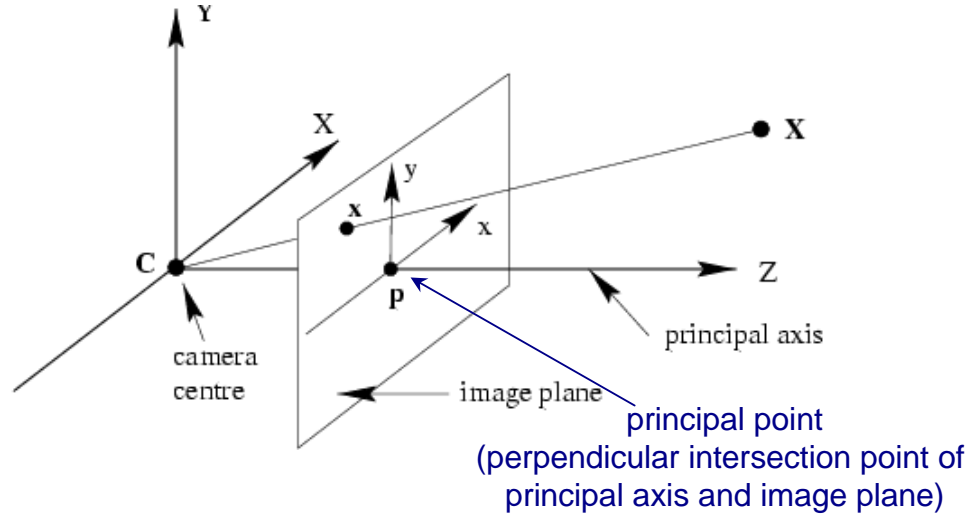
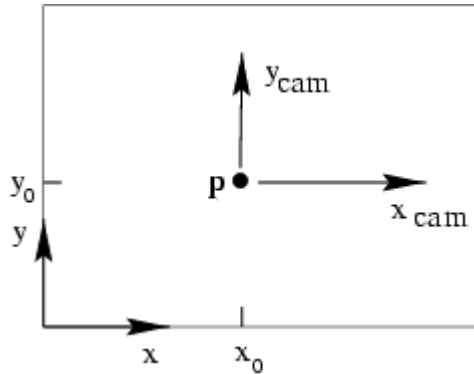


$$\begin{pmatrix} fX \\ fY \\ Z \end{pmatrix} = \begin{bmatrix} f & & & \\ & f & & \\ & & 1 & \\ & & & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

$$\mathbf{x} = \mathbf{P}\mathbf{X}$$

$$\mathbf{P} = \text{diag}(f, f, 1) [\mathbf{I} \mid \mathbf{0}]$$

Principal Point Offset



$$(X, Y, Z)^T \mapsto (fX/Z + p_x, fY/Z + p_y)^T$$

where $(p_x, p_y)^T$

are the coordinates of the principal point

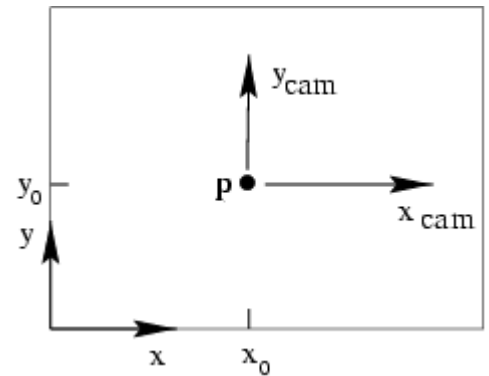
Principal Point Offset

$$\begin{pmatrix} fX + Zp_x \\ fY + Zp_y \\ Z \end{pmatrix} = \begin{bmatrix} f & 0 & 0 \\ 0 & f & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}$$

$$\mathbf{x} = \mathbf{K}[\mathbf{I} \mid \mathbf{0}]\mathbf{X}$$

where $\mathbf{K} = \begin{bmatrix} f & 0 & p_x \\ 0 & f & p_y \\ 0 & 0 & 1 \end{bmatrix}$

is called camera calibration matrix



Camera Rotation and Translation

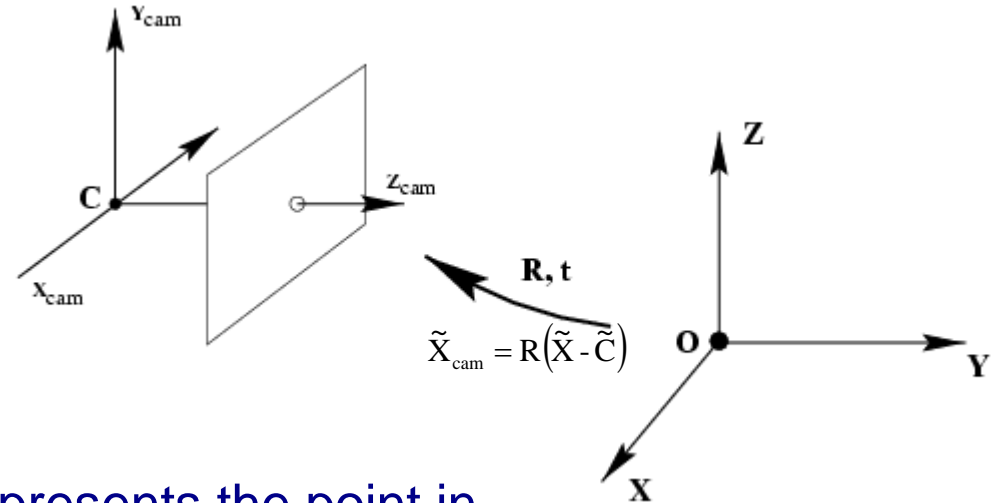
inhomogeneous coordinates

$$\tilde{\mathbf{X}}_{\text{cam}} = \mathbf{R}(\tilde{\mathbf{X}} - \tilde{\mathbf{C}})$$

where $\tilde{\mathbf{X}} = \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}$

$$\tilde{\mathbf{X}}_{\text{cam}} = \begin{pmatrix} X_{\text{cam}} \\ y_{\text{cam}} \\ Z_{\text{cam}} \end{pmatrix}$$

$$\tilde{\mathbf{C}}$$



represents the point in world coordinates

represents the same point in camera coordinates

represents the coordinates of the camera origin in the world coordinate frame

Extrinsic and Intrinsic Parameters

$$x = PX$$

where

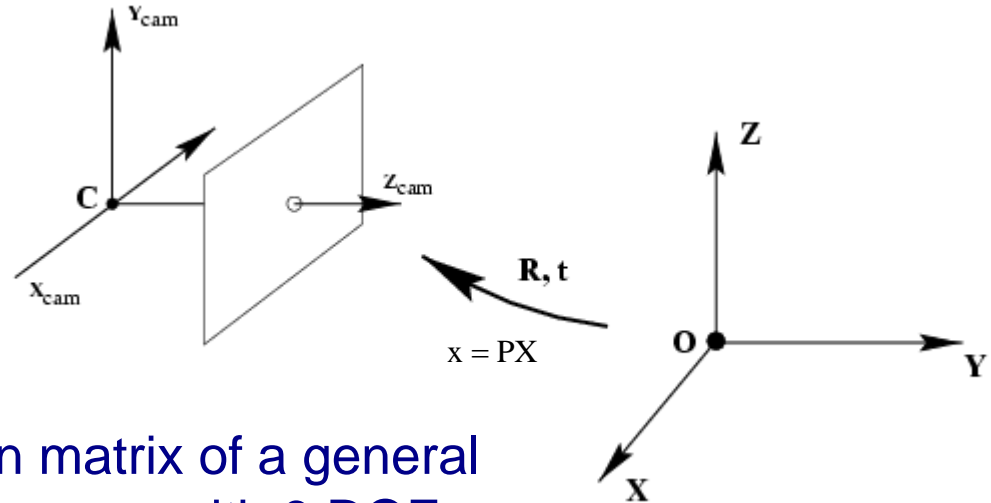
$$P = KR \left[I \mid -\tilde{C} \right]$$

projection matrix of a general pinhole camera with 9 DOF

$$K = \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix}$$

intrinsic camera parameters with 3 DOF

R, \tilde{C} extrinsic camera parameters with each 3 DOF
(camera orientation and position in world coordinates)



Skew Parameter

S skew parameter

$$K = \begin{bmatrix} \alpha_x & s & p_x \\ & \alpha_y & p_y \\ & & 1 \end{bmatrix} \quad 5 \text{ DOF}$$

$$P = KR \begin{bmatrix} I & | & -\tilde{C} \end{bmatrix} \quad \text{finite projective camera with 11 DOF}$$

Finite Projective Camera :: Summary

$$\mathbf{x} = \mathbf{P}\mathbf{X}$$

$$\mathbf{P} = \begin{bmatrix} \alpha_x & s & p_x \\ & \alpha_y & p_y \\ & & 1 \end{bmatrix} \begin{bmatrix} \mathbf{R} & -\mathbf{R}\tilde{\mathbf{C}} \\ 0 & 1 \end{bmatrix}$$

projection matrix
11 DOF (5+3+3)

$$= \underbrace{\mathbf{K}\mathbf{R}}_{\text{non-singular}} \begin{bmatrix} \mathbf{I} & | & -\tilde{\mathbf{C}} \end{bmatrix}$$

non-singular

Camera Anatomy

- camera center
- column vectors
- principal plane
- axis plane
- principal point
- principal ray

