

Computer Aided Medical Procedures II

Graph Cuts Segmentation

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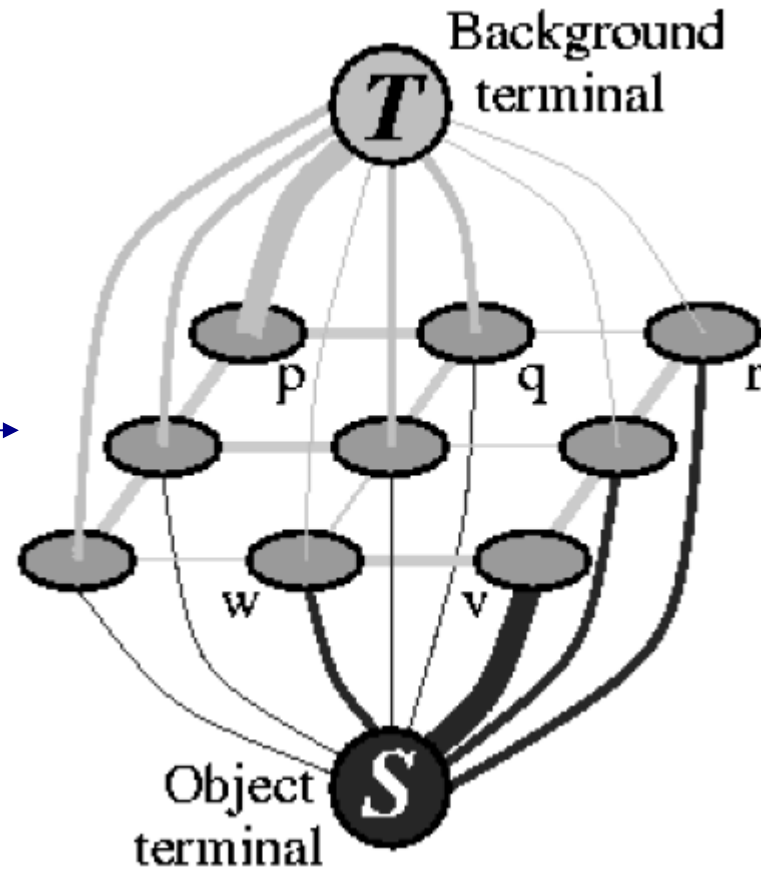
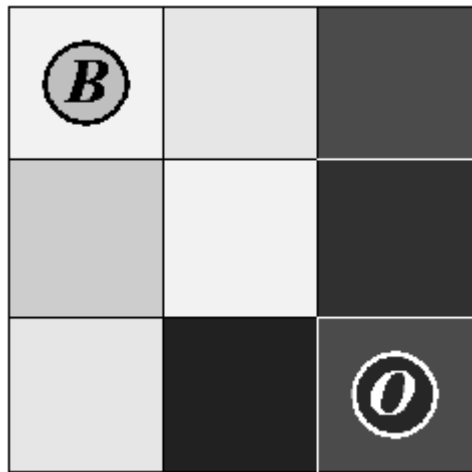
Graph-Cuts: Basic Idea – Build a graph

- We define a graph (undirected)

$$G = \langle V, E \rangle$$

- The nodes represent image pixels / voxels
- Two special nodes S (source) and T (sink) that represent “object” and “background” labels: *terminals*
- Neighboring pixels are interconnected by edges: *n-links*
- Another type of edges are used to connect pixels to terminals: *t-links*
- All edges are assigned some nonnegative weight (cost): w_e

Graph-Cuts: Basic Idea – Build a graph



Graph-Cuts: Basic Idea – Find a cut

- A s - t cut is a subset of edges $C \in V$ such that the terminals S and T become completely separated on the induced graph

$$G(C) = \langle V, E \setminus C \rangle$$

- Any cut corresponds to some binary partitioning of an image into “object” and “background”
- The goal is to compute the best cut that gives an “optimal” segmentation (e.i. a cut with minimal cost)

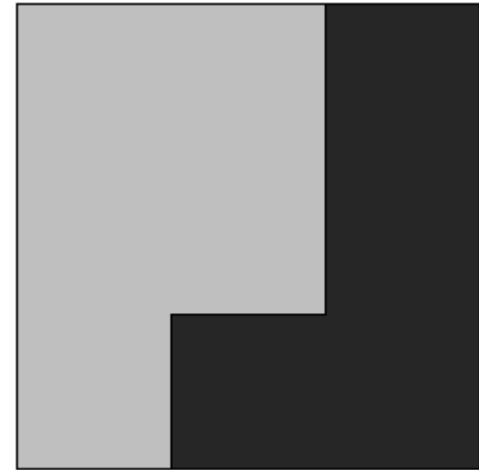
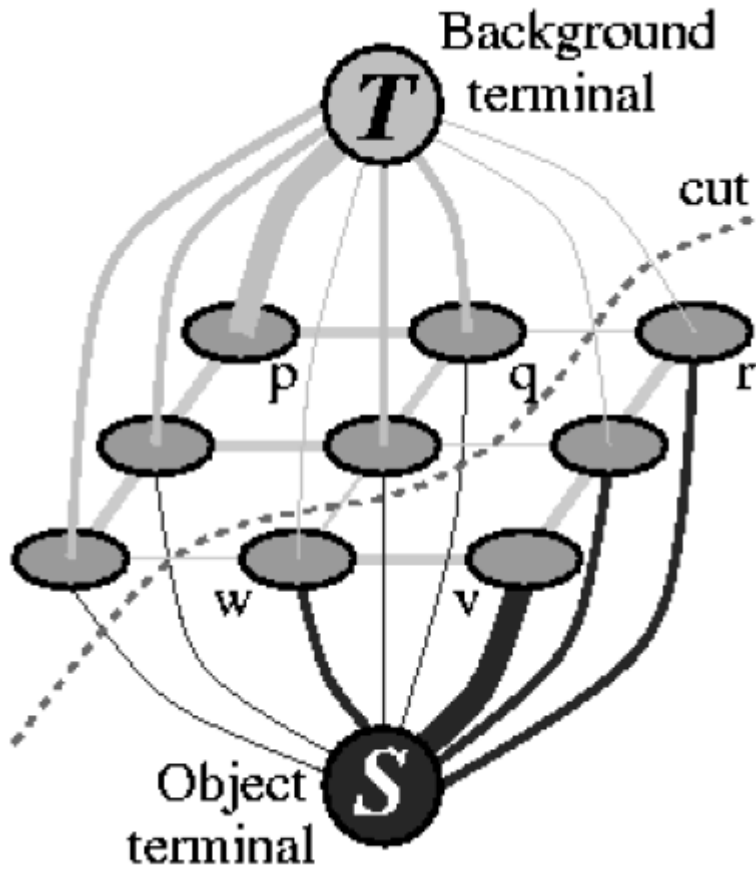
Graph-Cuts: Basic Idea – Find a cut

- Define the cost of a cut

$$|C| = \sum_{e \in C} w_e$$

- *n-links*: cost of segmentation **boundary**
- *t-links*: cost of **regional** properties
- infinity cost *t-links* make it possible to impose **hard constraints**
- A minimum cost cut may correspond to segmentation with desirable balance of **boundary** and **regional** properties
- Based on well-known combinatorial optimization of computing a minimum *s-t* cut (e.g. via max-flow or push-relabel)

Graph-Cuts: Basic Idea – Find a cut



Segmentation Energy

- Define the cost function

$$E(A) = \lambda \cdot R(A) + B(A)$$

- where

$$R(A) = \sum_{p \in P} R_p(A_p) \quad (\text{regional term})$$

$$B(A) = \sum_{\{p,q\} \in N} B_{p,q} \cdot \delta_{A_p \neq A_q} \quad (\text{boundary term})$$

- and

$$\delta_{A_p \neq A_q} = \begin{cases} 1 & \text{if } A_p \neq A_q \\ 0 & \text{if } A_p = A_q \end{cases}$$

Segmentation vector $A = (A_1, \dots, A_p, \dots, A_{|P|})$ A_p specifies assignment to pixels $p \in P$ (“obj” or “bkg”)

Neighborhood system by a set of N of all pairs $\{p, q\}$. Coefficient $\lambda \geq 0$ specifies region importance.

“Region” vs. “Boundary” – Soft constraints

- The regional term $R(A)$ assumes that the individual penalties for assigning pixel p to “object” and “background” are given.
 - For example: negative log-likelihoods
motivated by Maximum A Posterior estimation of a Markov Random Field (MAP-MRF)

$$R_p(\text{"obj"}) = -\ln \Pr(I_p \mid \text{"obj"})$$

$$R_p(\text{"bkg"}) = -\ln \Pr(I_p \mid \text{"bkg"})$$

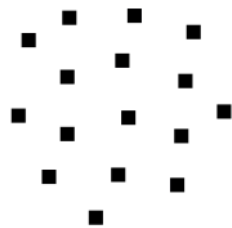
- A coefficient $B_{p,q} > 0$ in the boundary term $B(A)$ should be interpreted as a penalty for a discontinuity between p and q . $B_{p,q}$ is large when p and q are similar (e.g. in intensity)

- For example:

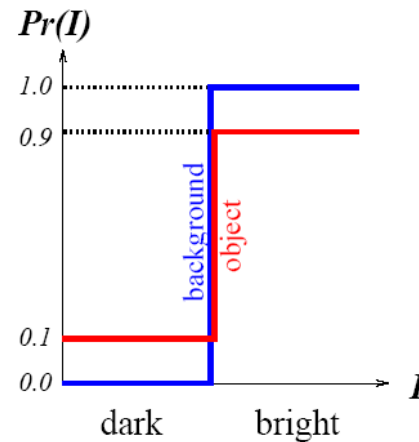
$$B_{p,q} \propto \exp\left(-\frac{(I_p - I_q)^2}{2\sigma^2}\right) \cdot \frac{1}{\text{dist}(p, q)}$$

“Region” vs. “Boundary” – Soft constraints

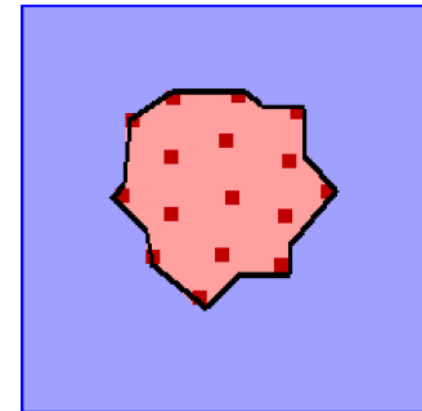
Example: Synthetic Gestalt



(a) Original image



(b) Intensity histograms



(c) Optimal segmentation

$$B_{p,q} = \begin{cases} 1 & \text{if } I_p = I_q \\ 0.2 & \text{if } I_p \neq I_q \end{cases}$$

I_p	$R_p(\text{“obj”})$	$R_p(\text{“bkg”})$
dark	2.3	$+\infty$
bright	0.1	0

Hard constraints

- Seed points (e.g. set by the user) / A priori knowledge
- Important source of “high level” contextual information

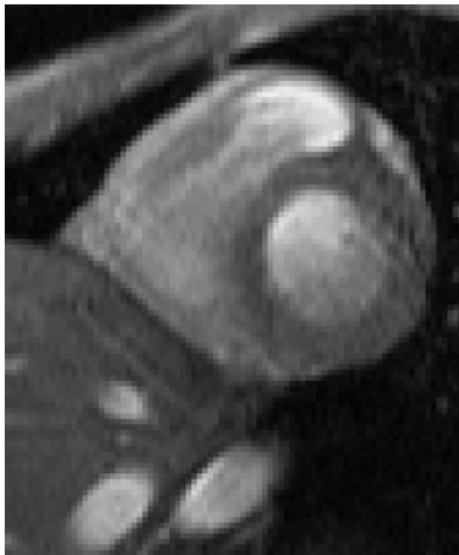
Pixel subsets O (“object”) and B (“background”) with $O \cap B = \emptyset$

$$\forall p \in O : A_p = \text{"obj"}$$

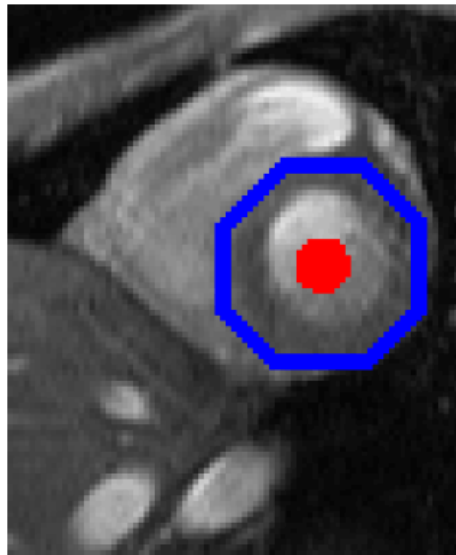
$$\forall p \in B : A_p = \text{"bkg"}$$

Hard constraints

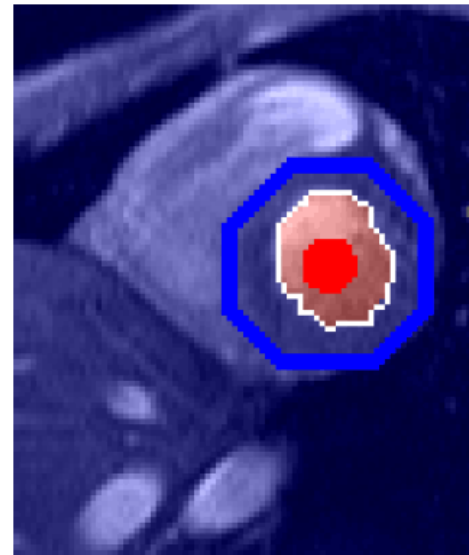
Example: Segmentation in medical image



(a) Original image

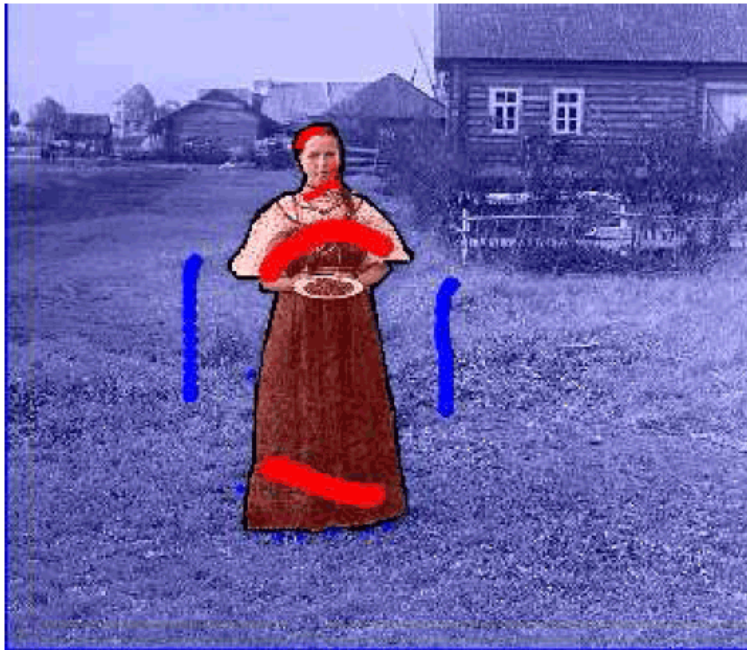


(b) Initialization

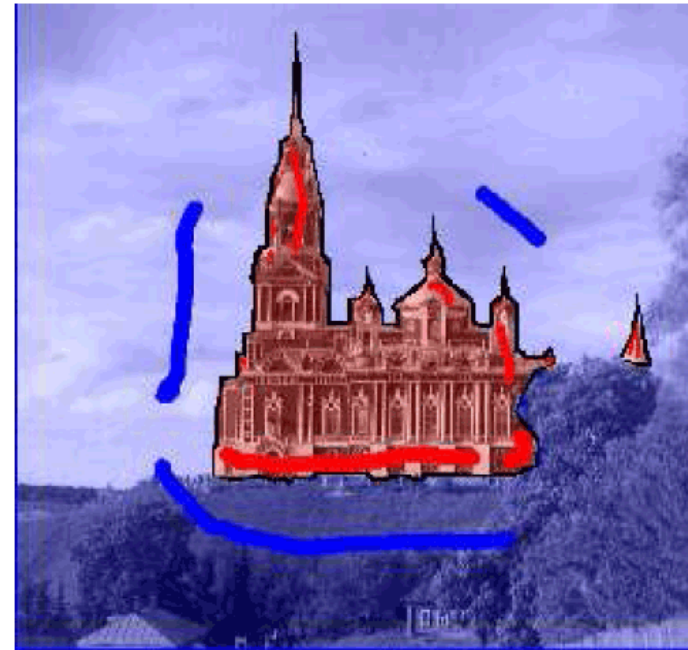


(c) Segmentation

Results with Graph-cuts



(a) A woman from a village



(b) A church in Mozhaisk (near Moscow)

Yuri Boykov, Gareth Funk-Lea. Optimal Object Extraction via Constrained Graph-Cuts. International Journal of Computer Vision, VISI 666-NP

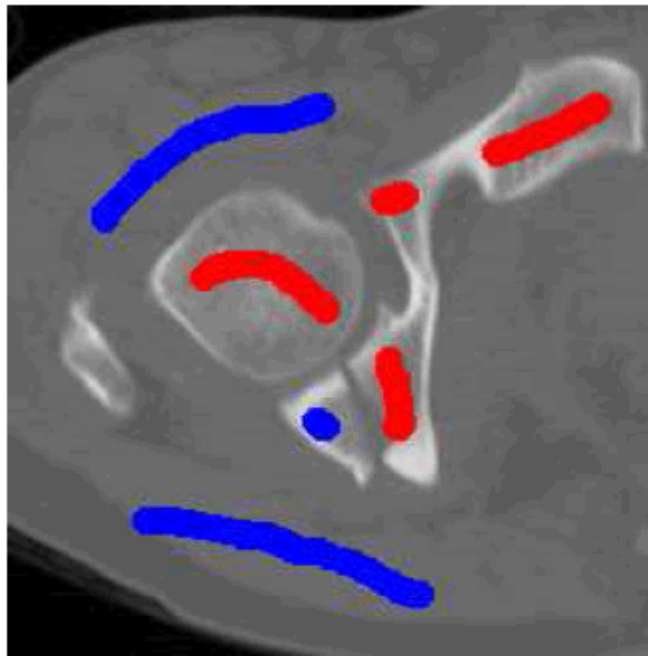
Results with Graph-cuts



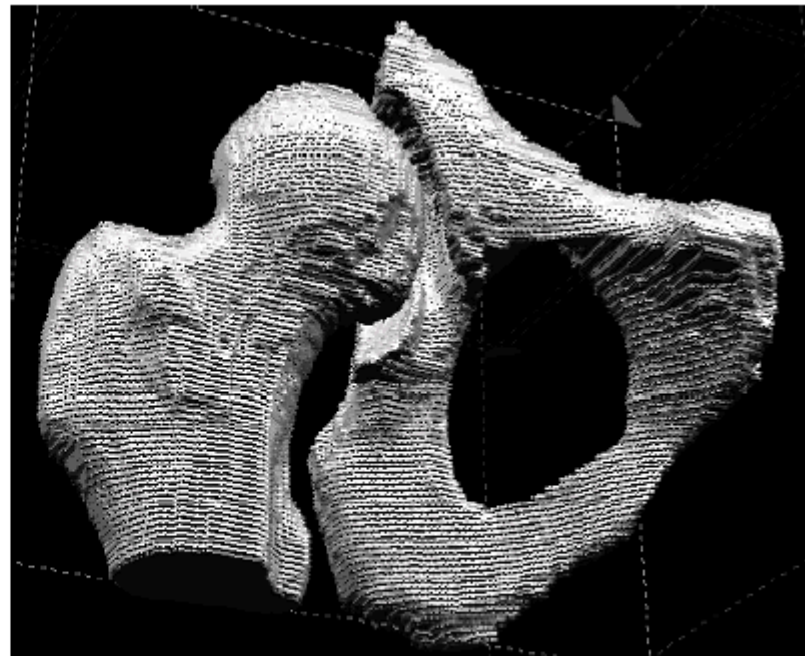
video sequence (21 frames)

Yuri Boykov, Gareth Funk-Lea. Optimal Object Extraction via Constrained Graph-Cuts. International Journal of Computer Vision, VISI 666-NP

Results with Graph-cuts



(a) A slice with seeds



(b) 3D object

Yuri Boykov, Gareth Funk-Lea. Optimal Object Extraction via Constrained Graph-Cuts. International Journal of Computer Vision, VISI 666-NP