



CAMP II: Intensity Based Registration

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June 19, 2007

Chair for Computer Aided Medical Procedures & Augmented Reality

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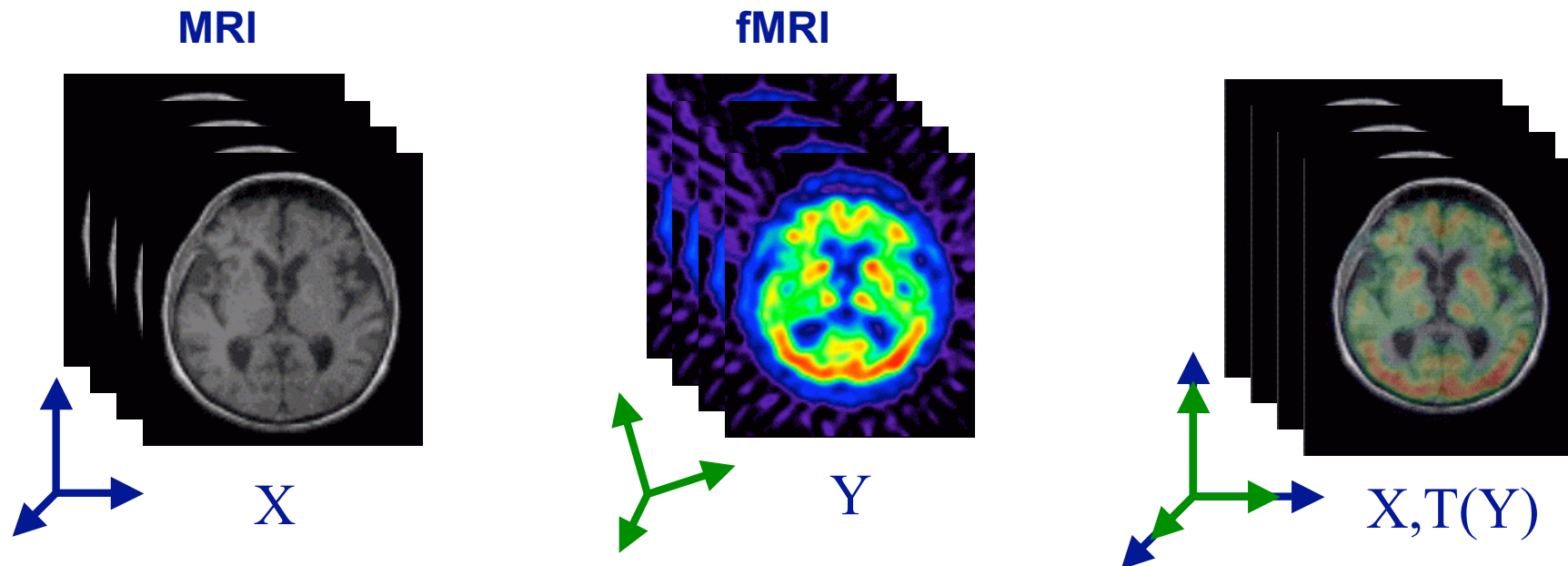
Outline

- Recall: Intensity Based Registration

- Similarity Measures
 - SSD/SAD
 - NCC
 - MI

- Practical Considerations
 - Interpolation
 - Optimizers

Registration

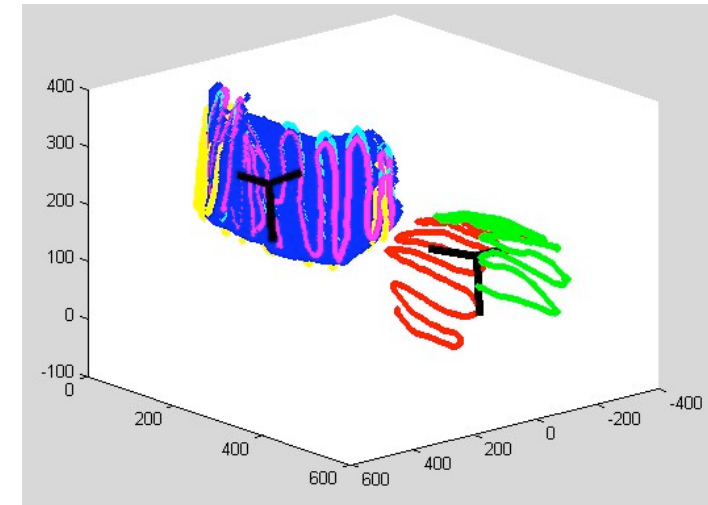


- Define Transformation T on one of the images
- Compare X , $T(Y)$ using either features or the full image content
- Refine T and repeat, until convergence criteria reached.

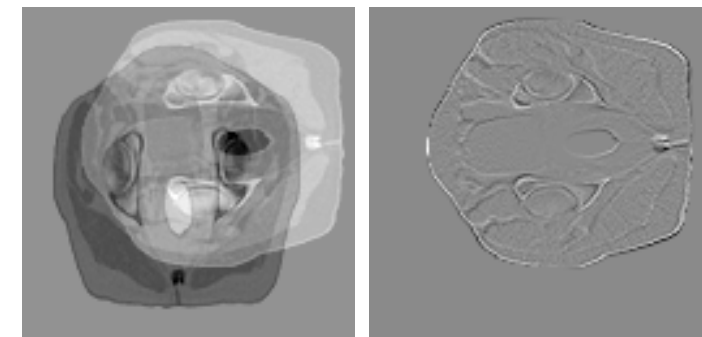
Registration Overview

- Feature-based methods
 - Point/line/curve/surface extraction
 - Registration of extracted entities only
 - Problem becomes one of **correspondences** and **distance**:
 - Corresponding Data available: Umeyama
 - No corresponding data available: PCA and ICP

- Intensity-based methods
 - Registration on pixel intensities only
 - Problem becomes one of **interpolation** and **similarity**:
 - How to get the right values to compare: Interpolation
 - How to compare the values in a meaningful way: Similarity Measures



Registration of point sets, see prev. lecture

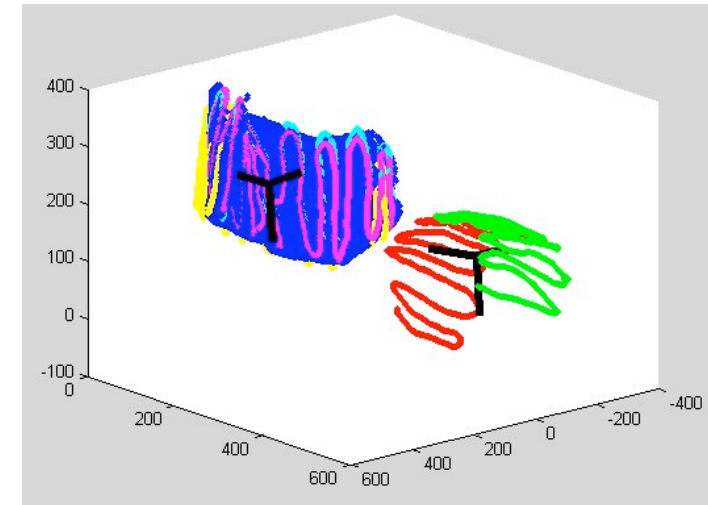


Registration of 2 CT slices, difference images

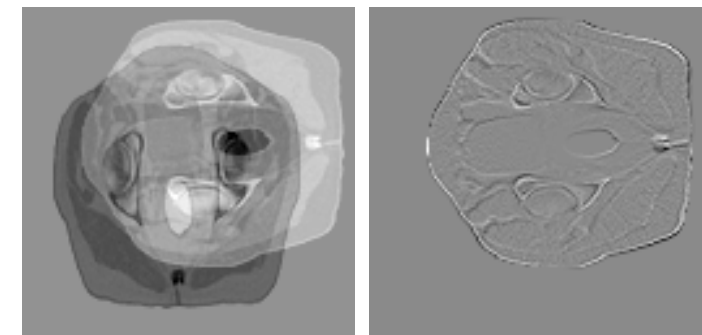
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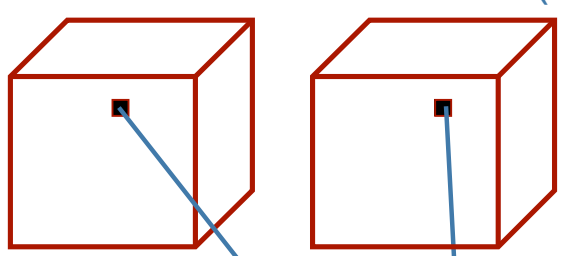
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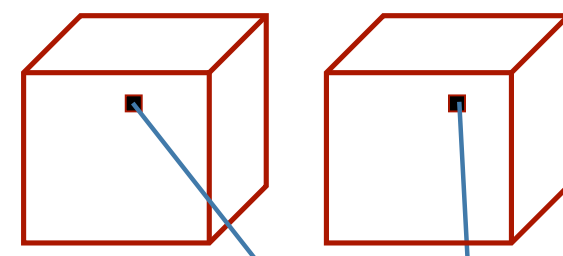
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Simple Similarity Measures: SSD / SAD

Volume X Volume T(Y)



Volume X Volume T(Y)



$$SSD = \frac{1}{N} \sum_i (x_i - y_i)^2$$

Sum of Squared Differences:
Simple measure, can be optimized with special algorithms

$$SAD = \frac{1}{N} \sum_i |x_i - y_i|$$

Sum of Absolute Differences:
Less sensitive on large intensity differences than SSD

>> Only for intramodality Registration, e.g. CT-CT!

Simple Similarity Measures (2)

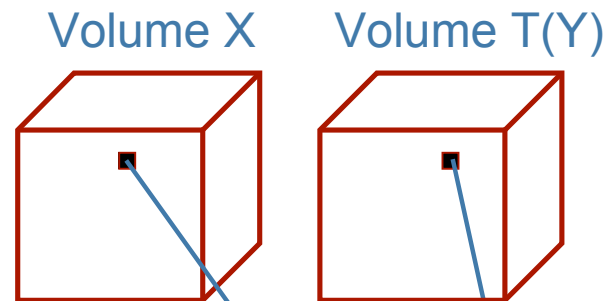
- Images slightly tilted:



- However: We encounter problems already if the images have different contrast or window/level values



Normalized Cross Correlation



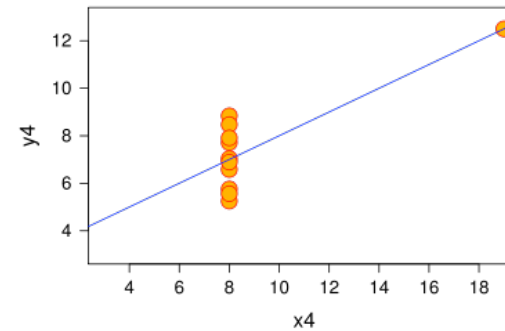
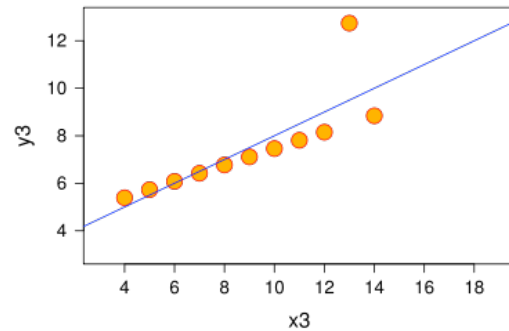
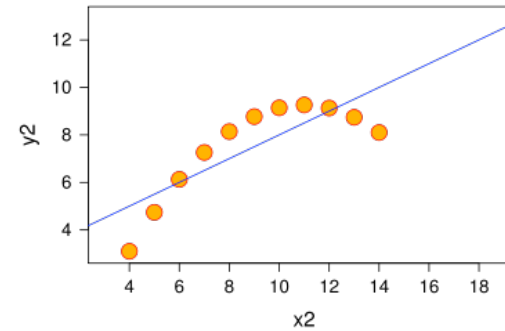
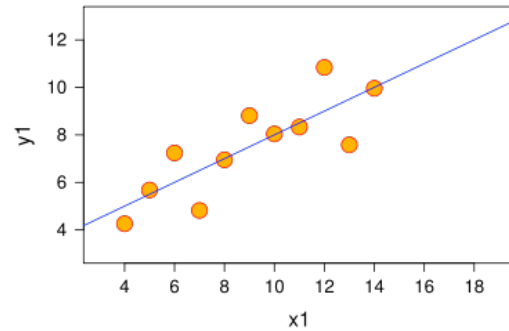
$$NCC = \frac{1}{\sigma_x \sigma_y} \sum_i (x_i - \bar{x})(y_i - \bar{y})$$

Normalized Cross Correlation:
Expresses the linear relationship between voxel intensities in the two volumes

→ **Already looser dependence on intensities!**

Normalized Cross Correlation - Example

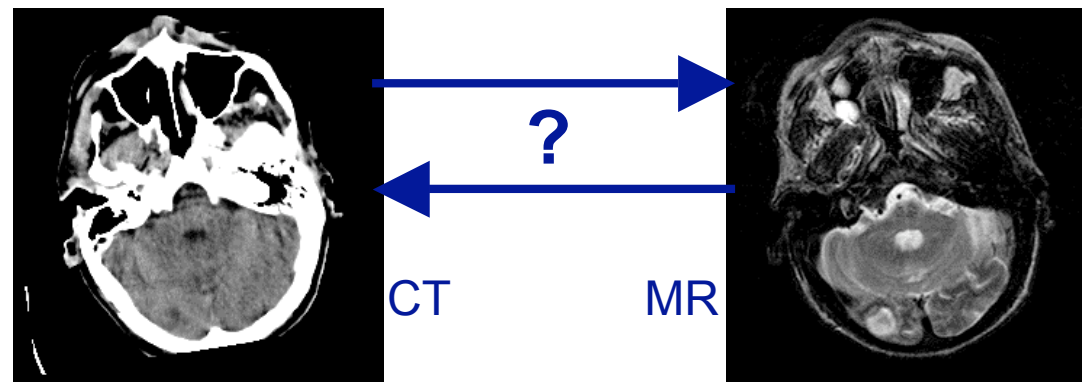
- Four datasets, two variables X,Y, $NCC(X,Y) = 0.81$ for each.



Source: wikipedia

Intensity-Based multimodal 3D-3D Registration

No simple relationship between X and Y intensities anymore:



Two approaches:

- Find an approximation for an intensity mapping from X to Y
- Use more sophisticated similarity measures

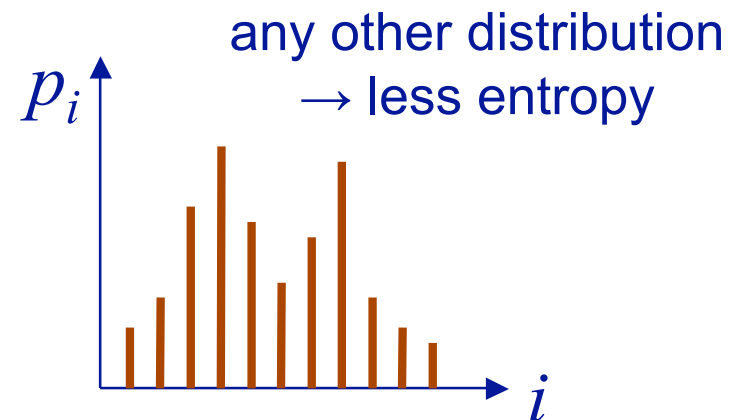
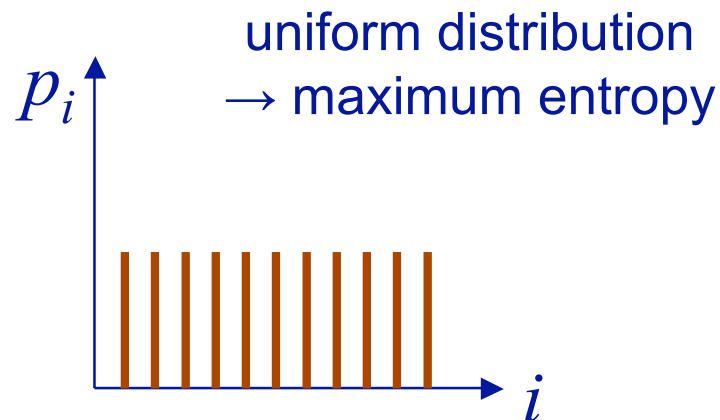
Information Theoretic Approach

- Registration means trying to maximize amount of information shared in the X and Y .
- An image combined from X , $T(Y)$ should have as little information as possible \gg all structures are aligned correctly.
- How to measure the amount of information?

Information Theoretic Approach - Entropy

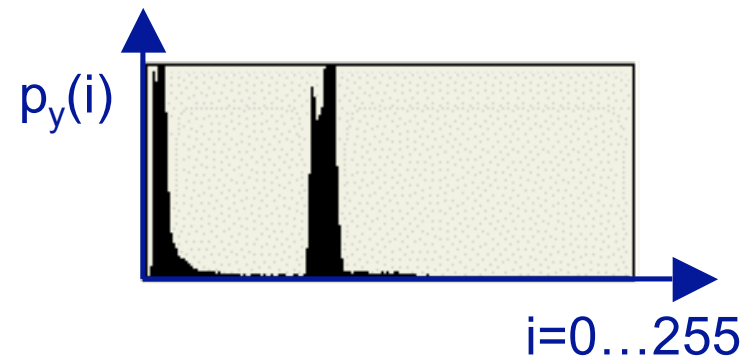
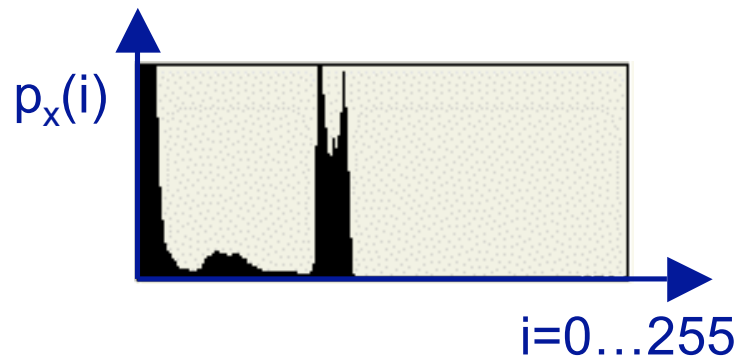
Shannon Entropy, developed in the 1940s
(information theory)

$$H = - \sum_i p_i \log p_i$$



Information Theoretic Approach - Entropy

- Images X and Y are treated as random variables
- $p_x(i)$ = probability of a pixel of X having the intensity value i
- $p_x(i)$ and $p_y(i)$ can be estimated from histograms



Joint Entropy

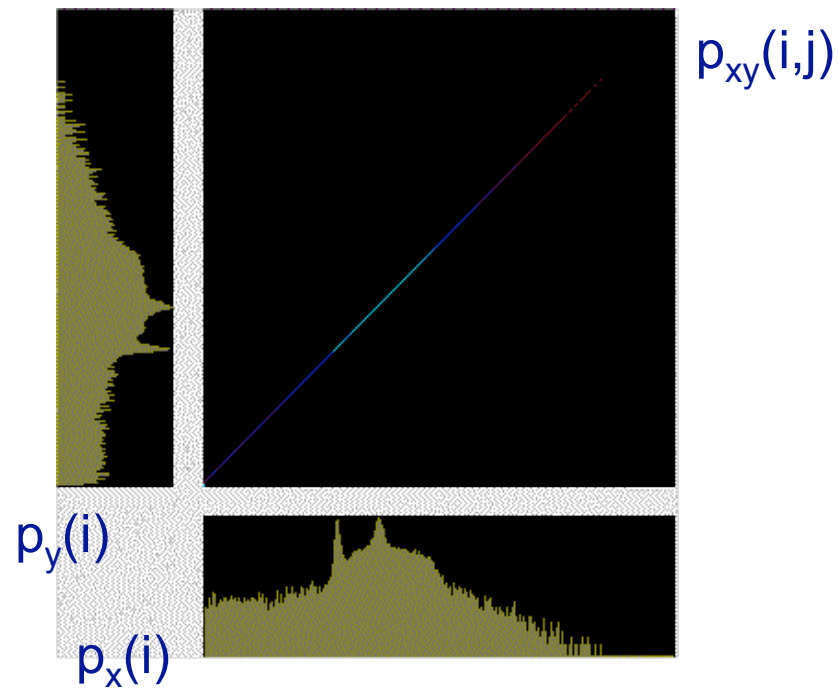
- At each voxel location, we have a pair of intensity values, representing the combined image.

$$H(X, Y) = - \sum_i \sum_j p_{xy}(i, j) \log p_{xy}(i, j)$$

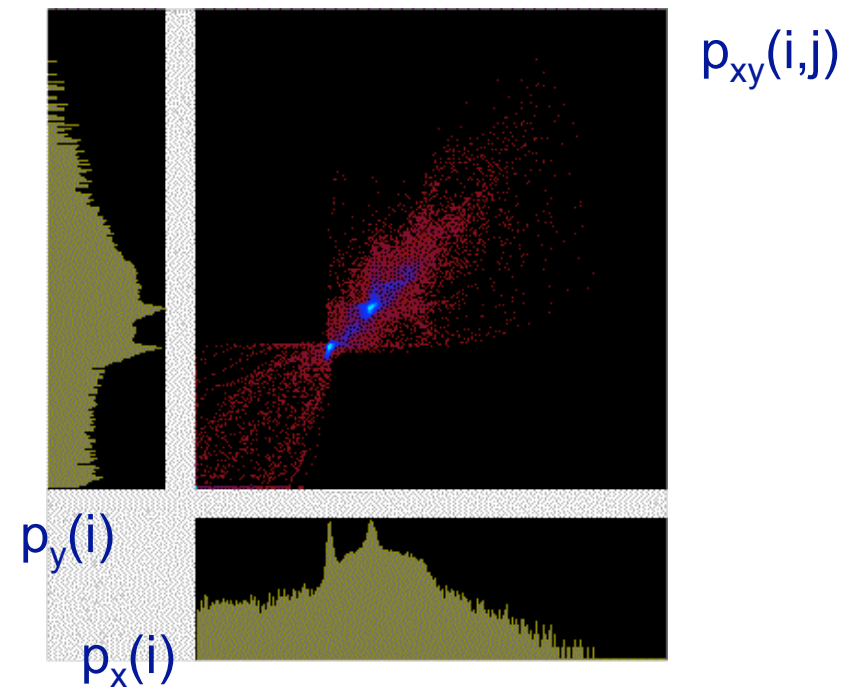
- X and Y totally unrelated: $H(X, Y) = H(X) + H(Y)$
- X and Y more similar
→ Joint Entropy smaller $H(X, Y) < H(X) + H(Y)$

Joint Entropy

X and Y identical



X and Y slightly misaligned



Mutual Information

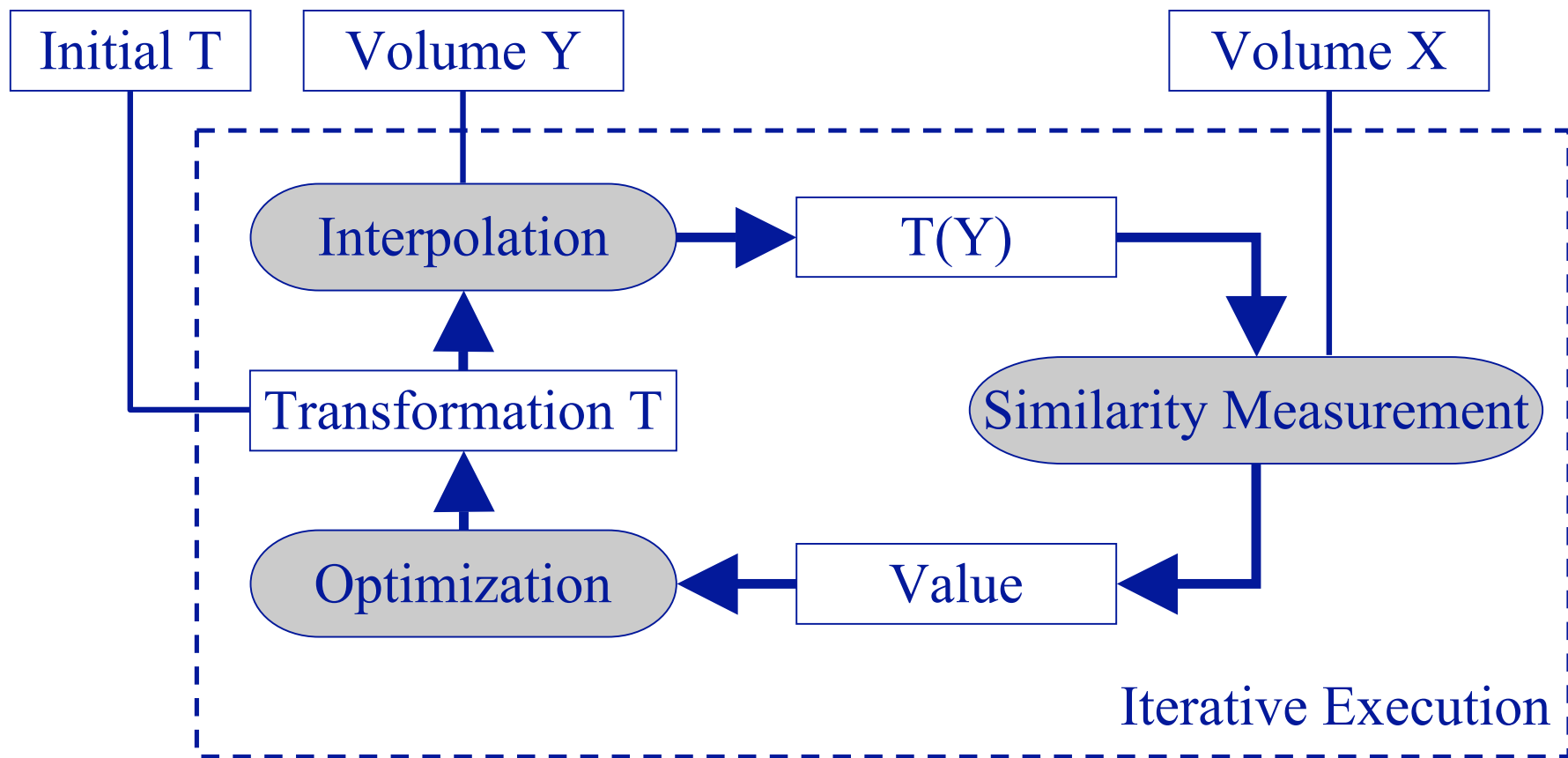
$$\begin{aligned} MI(X, Y) &= H(X) + H(Y) - H(X, Y) = \\ &= \sum_i \sum_j p_{xy}(i, j) \log \frac{p_{xy}(i, j)}{p_x(i)p_y(j)} \end{aligned}$$

- Maximized if X and Y are perfectly aligned
- Any permutation on the intensity values of X or Y does not affect the measure → great for multimodal registration!

- X and Y totally unrelated: $H(X, Y) = H(X) + H(Y)$

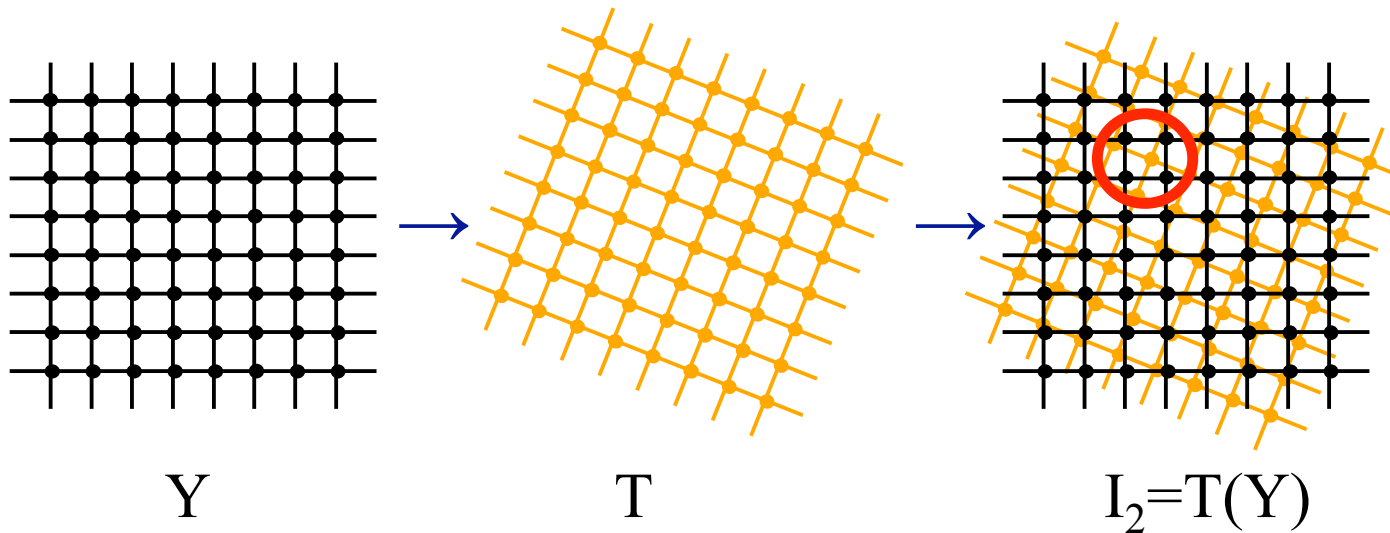
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Intensity-Based 3D-3D Registration



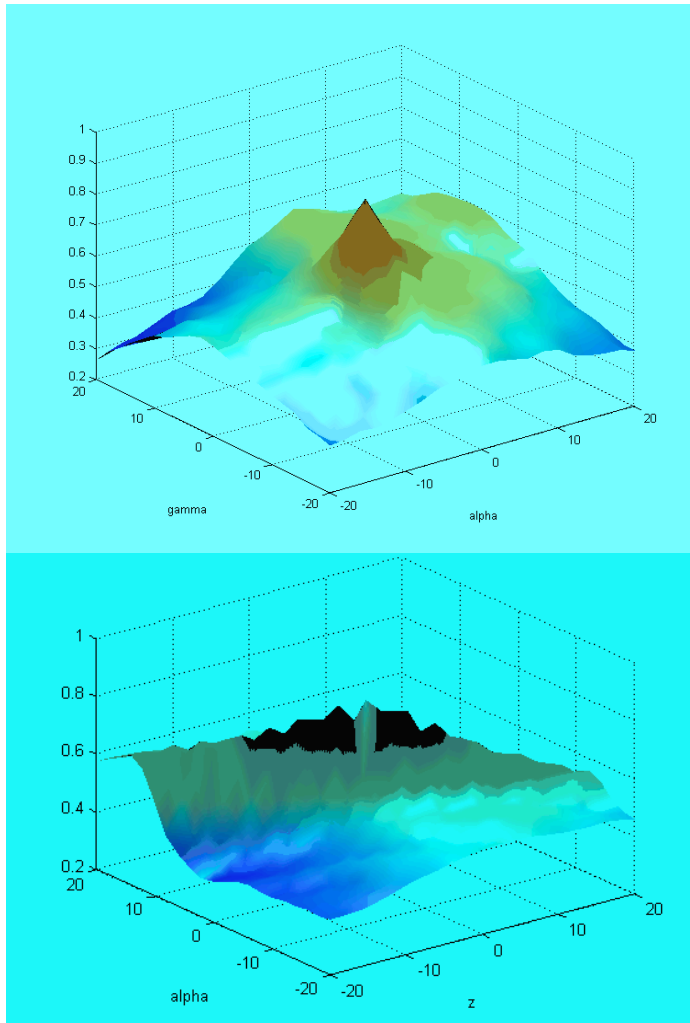
Practical Considerations: Interpolation

- Problem: applying transformation T on image Y while keeping same sampling grid



- Solution: Interpolation (nearest neighbor, bilinear..)

Practical Considerations: Optimization



- Similarity measure needs to be minimized / maximized, $f: \mathbb{R}^n \rightarrow \mathbb{R}$
- Direct search: Simplex, Best Neighbor, Powell-Brent
- Gradient information available: Gradient Descent, Steepest Descent, Newton

Conclusion

- Intensity Based Registration only dependent on pixel intensities (no detection of features needed)
- Very generic approach of iteratively refining the registration
- Choice of Similarity Measure determines characteristics of registration
 - Intra-modality registration: SSD/SAD, NCC
 - Inter-modality registration: joint entropy, MI
 - and many more...
- Interpolation crucial to minimize distortions / errors when applying transform
- Optimizer determines correctness (local minima) and convergence speed

Literature

- [1] Hajnal, Hill, Hawkes – *Medical Image Registration*
- [2] Maintz, Viergever - *An Overview of Medical Image Registration Methods*
- [3] Penney, Weese, Little, Desmedt, Hill, Hawkes – *A Comparison of Similarity Measures for Use in 2D-3D Medical Image Registration*, Lecture Notes in Computer Science, 2001
- [4] Pluim, Maintz, Viergever - *Mutual information based registration of medical images: a survey*, IEEE transactions on medical imaging, 2003
- [5] Viola - *Alignment by Maximization of Mutual Information*, PhD thesis, Massachusetts Institute of Technology, 1995.
- [6] Studholme, Hill, Hawkes - *An overlap invariant entropy measure of 3D medical image alignment*, Pattern Recognition, 1998