

Exercises in 3D Computer Vision

**Exercise 30 Self-calibration of the distortion of a zooming camera**

This exercise presents a new method for the self-calibration of the lens distortion of a zooming camera which appears for short focal lengths. The method is published in:

S. Benhimane and E. Malis, **Self-calibration of the distortion of a zooming camera by matching points at different resolutions** in *IEEE/RSJ International Conference on Intelligent Robots and Systems*, vol. 2, pp. 2307-2312, Sendai, Japan, October 2004.

The proposed technique does not need any special calibration pattern nor any prior knowledge about the environment. The key idea is to match points between a distorted image and an undistorted image taken at different resolutions. Matched points are used to compute invariants to the pinhole camera parameters. Then, lens distortion parameters are estimated in order to obtain the same invariants in both images. This approach is well suited to autonomous robotic vision applications. In fact, the self-calibration of the camera is done before moving the robot.

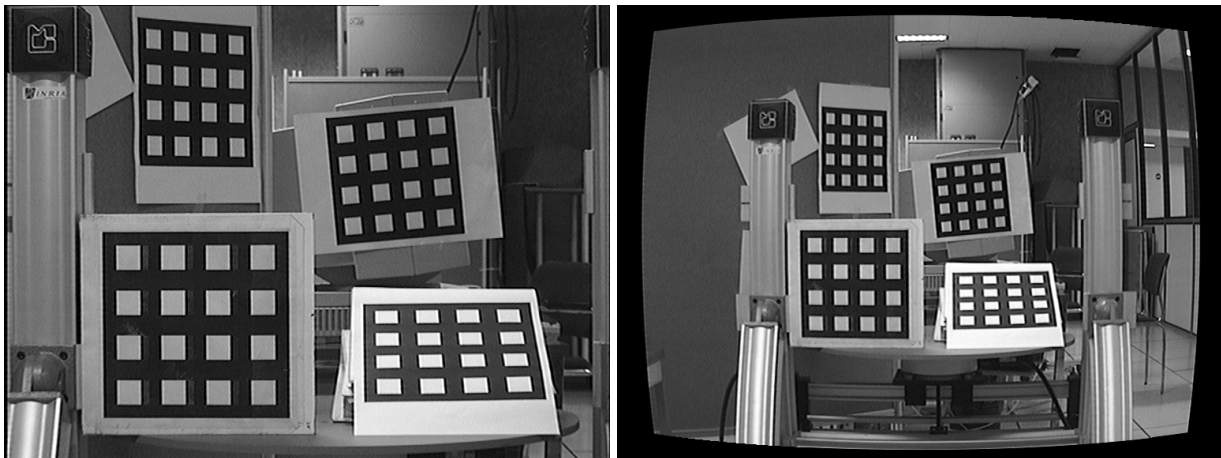


Figure 1: Undistorted image (with long focal length) and the distorted image (with short focal length)

Let  $(X_1, X_2, \dots, X_n)$  be 3D points belonging to a certain object, where  $(X_j, Y_j, Z_j)$  are the coordinates of  $X_j$  expressed in a absolute frame  $(C, \vec{i}, \vec{j}, \vec{k})$ . Suppose that we have a zooming camera observing the object. Let  $\mathbf{K}_i$  be the camera intrinsic parameters at the moment of the capture of the image  $i$ :

$$\mathbf{K}_i = \begin{bmatrix} f_i & 0 & u_{0i} \\ 0 & f_i & v_{0i} \\ 0 & 0 & 1 \end{bmatrix}$$

Let  $\mathbf{m}_j = [x_j, y_j, 1]^T$  be the projection of the 3D point  $X_j$  on the plane  $\pi$  perpendicular to the optical axis  $\vec{k}$  and verifying:  $distance(C, \pi) = 1$ , and let  $\mathbf{p}_{ji} = [u_{ji}, v_{ji}, 1]^T$  be its projection in the image  $i$  ( $u_{ji}$  and  $v_{ji}$  are in pixels).

- a) Give the expression of  $x_j$ ,  $y_j$ ,  $u_{ji}$  and  $v_{ji}$ .
- b) Let  $\mathbf{S}_{pi} = \frac{1}{n} \sum_{j=1}^n \mathbf{p}_{ji} \mathbf{p}_{ji}^\top$  and  $\mathbf{S}_m = \frac{1}{n} \sum_{j=1}^n \mathbf{m}_j \mathbf{m}_j^\top$ . Express  $\mathbf{S}_{pi}$  and  $\mathbf{S}_m$  using  $x_j$ ,  $y_j$ ,  $u_{ji}$  and  $v_{ji}$ . What is the geometric meaning of the entries of the matrices  $\mathbf{S}_{pi}$  and  $\mathbf{S}_m$ .
- c) Give the equation that links  $\mathbf{S}_{pi}$  and  $\mathbf{S}_m$ .
- d) Show that  $\mathbf{S}_{pi}$  and  $\mathbf{S}_m$  are not definite positive for  $n < 3$ .

A  $(3 \times 3)$  matrix  $\mathbf{M}$  is definite positive iff  $\forall \mathbf{x} \in \mathbb{R}^3 \neq \mathbf{0}_{n \times 1}$ , we have  $\mathbf{x}^\top \mathbf{M} \mathbf{x} > 0$ .

We can also say that a symmetric matrix  $\mathbf{M} = \begin{bmatrix} a & b & c \\ b & d & e \\ c & e & f \end{bmatrix}$  is definite positive iff the leading principal minors have a positive determinant :  $a > 0$  and  $ad - b^2 > 0$  and  $\det(\mathbf{M}) > 0$  (the Sylvester criterion).

- e) Let  $\mathbf{x} \in \mathbb{R}^n$ . Show that we have:

$$\mathbf{x}^\top \mathbf{x} = 0 \text{ iff } \mathbf{x} = \mathbf{0}_{n \times 1}$$

For  $n \geq 3$  non-colinear points, demonstrate that  $\mathbf{S}_p$  and  $\mathbf{S}_m$  are definite positive matrices.

- f) Let  $\mathbf{T}$  be the (well defined) following matrix:

$$\mathbf{T} = \begin{bmatrix} \sqrt{\sigma_{11} - \sigma_{13}^2 - \frac{(\sigma_{12} - \sigma_{13}\sigma_{23})^2}{\sigma_{22} - \sigma_{23}^2}} & \frac{\sigma_{12} - \sigma_{13}\sigma_{23}}{\sqrt{\sigma_{22} - \sigma_{23}^2}} & \sigma_{13} \\ 0 & \sqrt{\sigma_{22} - \sigma_{23}^2} & \sigma_{23} \\ 0 & 0 & 1 \end{bmatrix}$$

Compute the product  $\mathbf{T} \mathbf{T}^\top$ . Find the expression of the matrices  $\mathbf{T}_{pi}$  and  $\mathbf{T}_m$  verifying  $\mathbf{S}_{pi} = \mathbf{T}_{pi} \mathbf{T}_{pi}^\top$  and  $\mathbf{S}_m = \mathbf{T}_m \mathbf{T}_m^\top$ .

- g) Give the relationship between  $\mathbf{T}_m$  and  $\mathbf{T}_{pi}$ .
- h) Show that the vectors  $\mathbf{q}_{ji} = \mathbf{T}_{pi}^{-1} \mathbf{p}_{ji} = \mathbf{q}_j$  are camera independent.
- i) We consider the standard radial distortion model, but any other parametric distortion model could be used. Let  $\mathbf{p}_{ci} = (u_{ci}, v_{ci}, 1)$  be the unknown center of radial distortion of the image  $i$  (not necessarily equal to the principal point), and let  $\rho_{ji}$  be the distance of a distorted point  $\mathbf{p}_{ji}^d = (u_{ji}^d, v_{ji}^d, 1)$  from  $\mathbf{p}_{ci}$ :

$$\rho_{ji} = \|\mathbf{p}_{ji}^d - \mathbf{p}_{ci}\| = \sqrt{(u_{ji}^d - u_{ci})^2 + (v_{ji}^d - v_{ci})^2}$$

The undistorted coordinates  $\mathbf{p}_{ji}$  of an image point can be computed as:

$$\mathbf{p}_{ji} = \mathbf{p}_{ji}^d + (\mathbf{p}_{ji}^d - \mathbf{p}_{ci})(d_{1i}\rho_{ji}^2 + d_{2i}\rho_{ji}^4 + \dots)$$

The camera distortion parameters of the image  $i$  are the center of radial distortion  $\mathbf{p}_{ci}$  and the vector  $\mathbf{d}_i = (d_{1i}, d_{2i}, \dots)$ .

Give a method that makes it possible to undistort short focal length images of a zooming camera.