# 3D Computer Vision - Exercises - WT 2005

## Reminder Conics, Quadrics, Quadratic Forms

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1. Quadratic Forms, Conics and Quadrics
2. Interpretation
3. Summary and References

# Quadratic Form

- We can express quadratic functions by any square matrix $A$:

$$f(x) = x^T A x$$

- It can be shown that we can assume that $A$ is symmetric without loss of generality
- Given a symmetric matrix $A$ then we denote to the expression $f(x) = x^T A x$ as Quadratic Form

# Conic

- A Conic is a curve described by a quadratic equation in a plane
- For homogeneous coordinates this is a set of all points $x \in \mathbf{R}^3$ which satisfy the equation:

$$0 = x^T C x$$

where $C$ is a symmetric $3 \times 3$ matrix

# Conic - Detailed

$$\text{For } x = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and } C = \begin{bmatrix} a & b/2 & d/2 \\ b/2 & c & e/2 \\ d/2 & e/2 & f \end{bmatrix}$$

We have

$$x^T C x = ax^2 + bxy + cy^2 + dxz + exz + fz^2$$

# Quadric

- A Quadric is a set of all points $x \in \mathbf{R}^4$ which satisfy the equation:

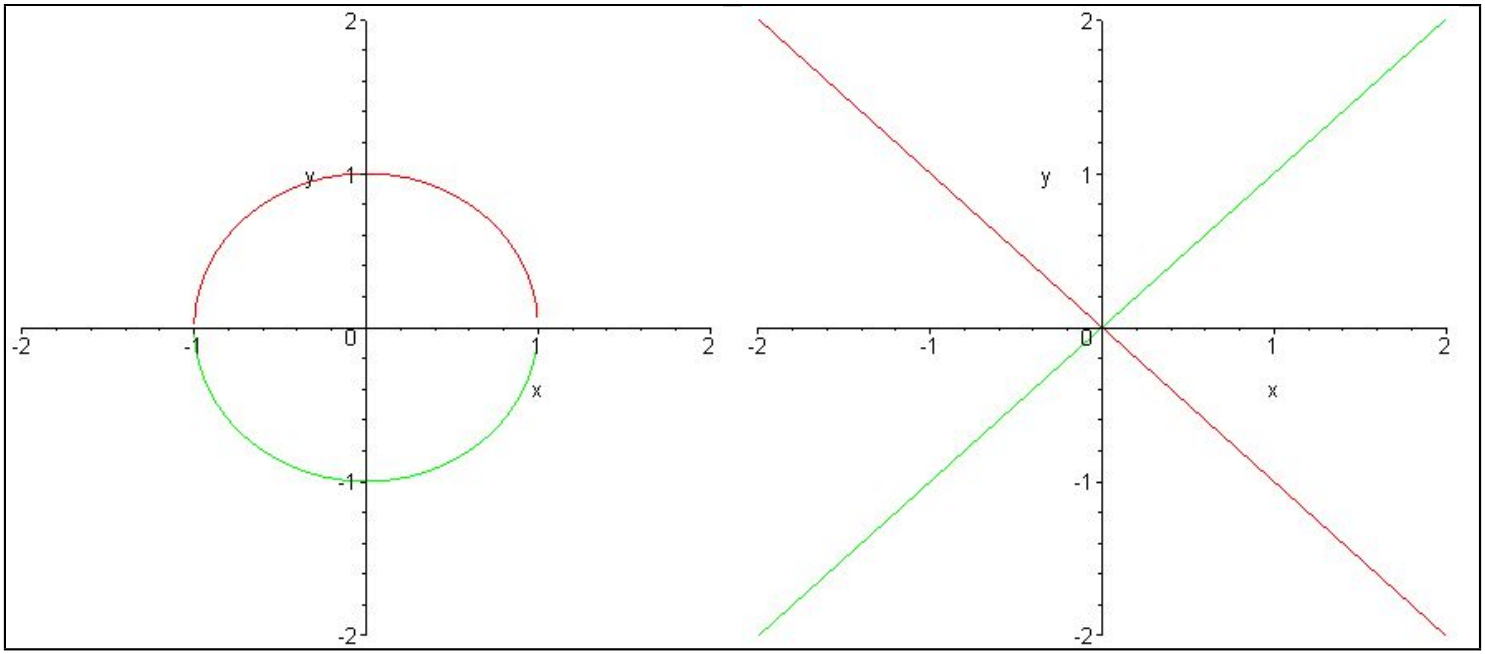
$$0 = x^T Q x$$

where $Q$ is a symmetric $4 \times 4$ matrix

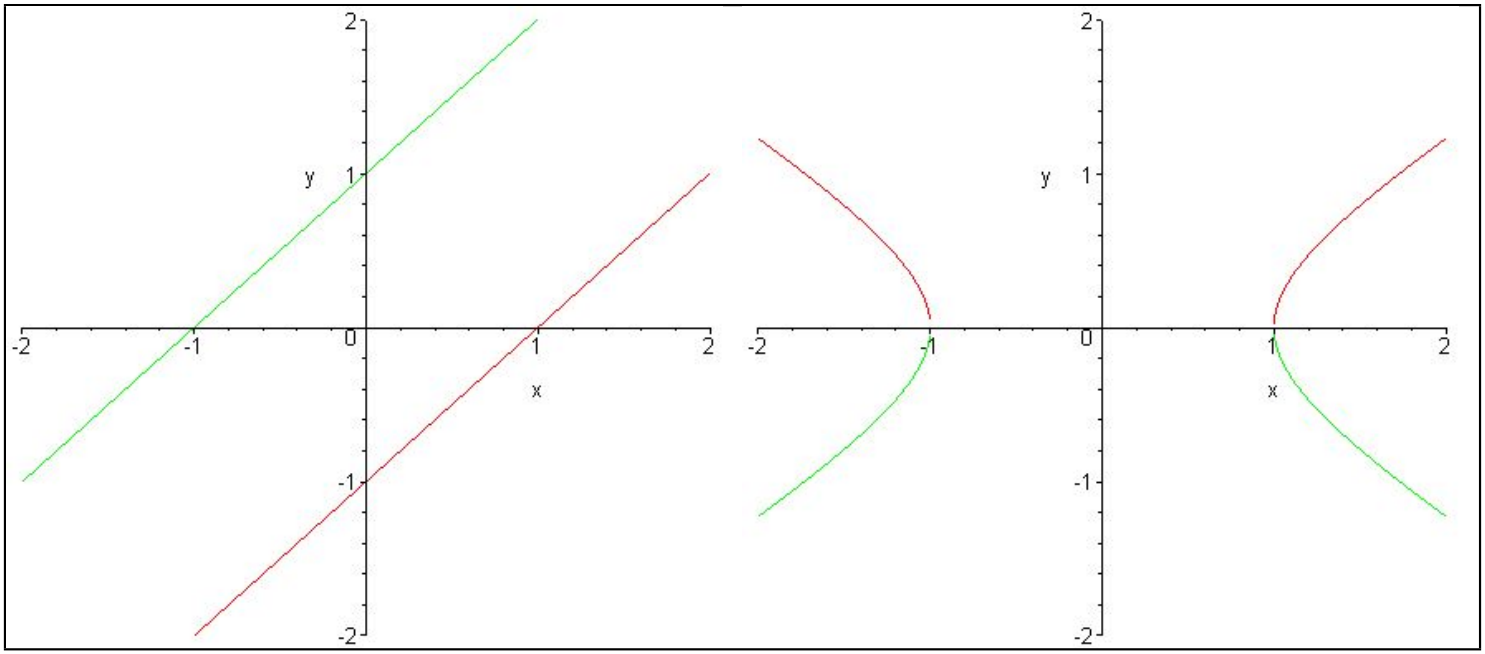
# Remarks

- The fact that the matrix $A$ in the quadratic form $f(x) = x^T A x$ is symmetric has some nice properties
- Given a symmetric matrix $A \in \mathbf{R}^{n \times n}$, we have:
  - All eigenvalues $\lambda_i$ of $A$ are real numbers ($\lambda_i \in \mathbf{R}$)
  - Eigenvectors belonging to different eigenvectors of $A$ are orthogonal
  - algebraic and geometric multiplicities are the same
- This makes it possible for us to transform any conic/quadric to a diagonal matrix

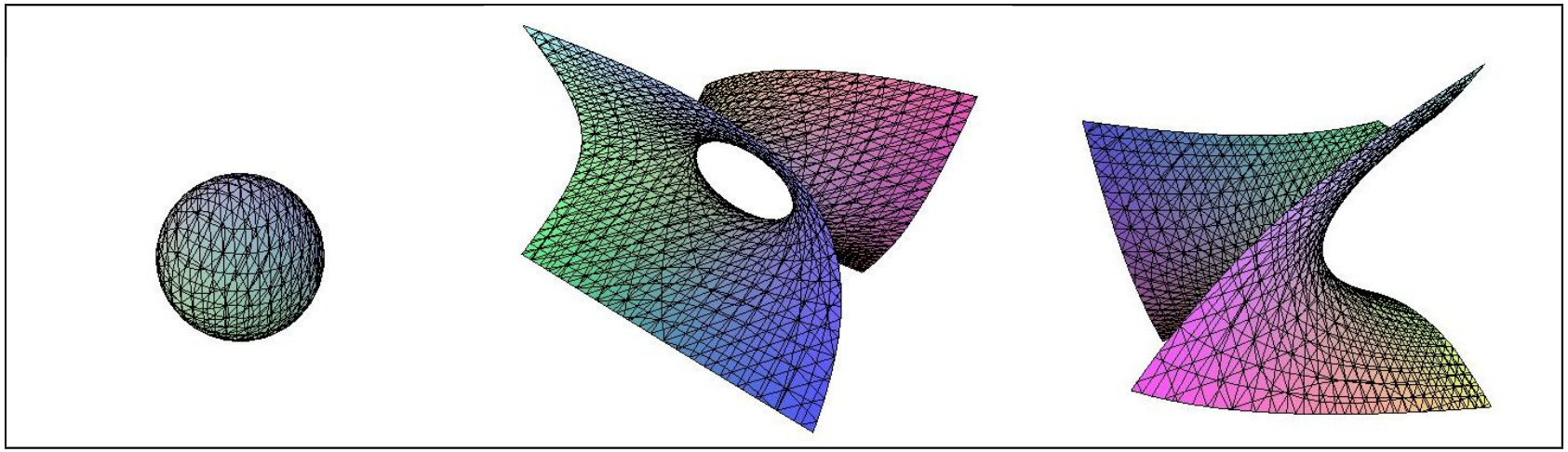
# Conic



# Conic



# Quadric



# References


- Hartley and Zisserman. *Multiple View Geometry* (p. 30, p. 73)
- Vachenauer. *Höhere Mathematik 1* (Chapter 6, Section 7)