

Exercises in 3D Computer Vision

Exercise 1 Line Fitting (Exemplary Solution)

see script chapter for parameter estimation

Exercise 2 Error Models (Exemplary Solution)

- a) Following notation is used: A point $\vec{x}_i = (x_i, y_i, w_i)^T$ transforms via the homography H to $\vec{x}'_i = (x'_i, y'_i, w'_i)$. The estimation vector $(\hat{x}'_i, \hat{y}'_i, \hat{w}'_i)^T = \vec{\hat{x}}'_i = Hx_i$ is defined. Then, we can write

$$A_i h = \begin{pmatrix} y'_i \vec{h}^{3T} \vec{x}_i - w'_i \vec{h}^{2T} \vec{x}_i \\ w'_i \vec{h}^{1T} \vec{x}_i - x'_i \vec{h}^{3T} \vec{x}_i \end{pmatrix} = \begin{pmatrix} y'_i \hat{w}'_i - w'_i \hat{y}'_i \\ w'_i \hat{x}'_i - x'_i \hat{w}'_i \end{pmatrix}$$

Thus, the algebraic distance is defined as

$$\|A_i h\| = \left((y'_i \vec{h}^{3T} \vec{x}_i - w'_i \vec{h}^{2T} \vec{x}_i)^2 + (w'_i \vec{h}^{1T} \vec{x}_i - x'_i \vec{h}^{3T} \vec{x}_i)^2 \right)^{1/2}$$

For points \vec{x}'_i and $\vec{\hat{x}}'_i$ the geometric distance is defined as

$$d(\vec{x}'_i, \vec{\hat{x}}'_i) = \left((x'_i/w'_i - \hat{x}'_i/\hat{w}'_i)^2 + (y'_i/w'_i - \hat{y}'_i/\hat{w}'_i)^2 \right)^{1/2} = \|A_i h\| / \hat{w}'_i w'_i$$

Thus, if $\hat{w}'_i = w'_i = 1$ the distances are identical and minimizing the algebraic error equals to minimizing the geometric error. However, this is not always the case, especially $\hat{w}'_i = 1$ since it depends on H . However, if H is an affinity, i.e. $\vec{h}^3 = (0, 0, 1)$, then assuring that $w'_i = 1$ automatically leads to $\hat{w}'_i = 1$.

- b) These are the “corrected measurement points”, i.e. the noise in the measured points is considered and they are altered such that they resemble more the true points. Thus, the reprojection error does not only consider a best homography between measured points, but determines a best homography of corrected (almost true) measured points.
- c) If the noise obeys a Gaussian distribution with zero mean and uniform standard deviation, then the distance from measured (noisy) point \vec{x}_i to true point $\vec{\hat{x}}_i$ is gaussian, i.e. the probability for a measured point \vec{x}_i to have distance $d(\vec{x}_i, \vec{\hat{x}}_i)$ from the real point $\vec{\hat{x}}_i$ is

$$Pr(\vec{x}_i) = \left(\frac{1}{2\pi\sigma^2} \right) e^{-d(\vec{x}_i, \vec{\hat{x}}_i)^2 / (2\sigma^2)}$$

Now, given a homography H , the probability of obtaining a measured point correspondence (\vec{x}_i, \vec{x}'_i) given that the true homography is H and the true point in the first image is $\vec{\hat{x}}'_i$ is

$$Pr((\vec{x}_i, \vec{x}'_i) | H, \vec{\hat{x}}'_i) = \frac{1}{2\pi\sigma^2} e^{-(d(\vec{x}_i, \vec{\hat{x}}_i)^2 + d(\vec{x}'_i, H\vec{\hat{x}}_i)^2) / (2\sigma^2)} =: Pr(i | H, \vec{\hat{x}}'_i).$$

The probabilities of obtaining the measurements of all point correspondences $\{\vec{x}_i, \vec{x}'_i\}$ given that the true homography is H and the true points in the first image are $\{\vec{x}_i\}$ is the product of all the individual probabilities since the measurement errors are assumed to be independent:

$$Pr(\{\vec{x}_i, \vec{x}'_i\} | H, \{\vec{x}_i\}) = \prod_i Pr(i | H, \vec{x}_i)$$

Now, the log-likelihood is determined by

$$\log Pr(\{\vec{x}_i, \vec{x}'_i\} | H, \{\vec{x}_i\}) = -\frac{1}{2\sigma^2} \sum_i (d(\vec{x}_i, \vec{x}_i)^2 + d(\vec{x}'_i, H\vec{x}_i)^2) + \text{constant}$$

Here, the parameters to be found are not only the homography entries H but also the true measurements \vec{x}_i . The MLE maximizes the log-likelihood, i.e. each step of maximisation defines a new optimal homography \hat{H} and new estimated measurements $\{\hat{x}_i \leftrightarrow \vec{x}'_i\}$ that play the role of the true measurements $\{\vec{x}_i \leftrightarrow \vec{x}'_i\}$, such that $\vec{x}'_i = \hat{H}\hat{x}_i$. Hence, the MLE maximizes

$$\log Pr(\{\vec{x}_i, \vec{x}'_i\} | H, \{\vec{x}_i\}) = -\frac{1}{2\sigma^2} \sum_i (d(\vec{x}_i, \hat{x}_i)^2 + d(\vec{x}'_i, H\hat{x}_i)^2) + \text{constant}$$

and therefore i.e. minimizes the function

$$\sum_i (d(\vec{x}_i, \hat{x}_i)^2 + d(\vec{x}'_i, H\hat{x}_i)^2),$$

which is the reprojection error.