

Exercises in 3D Computer Vision

Exercise 25 Harris Corner Detector (Exemplary Solution)

- a) Let the original input image be I and the modified input image $I' = \alpha I + \beta$. During the calculation of the Harris corner detector, first the image derivatives I_x and I_y are calculated. The modified derivatives are $I'_x = \alpha I_x$ and $I'_y = \alpha I_y$. The next step is to create the images I_x^2 , $I_x I_y$ and I_y^2 . This gives $I'^2_x = \alpha^2 I_x^2$, $I'_x I'_y = \alpha^2 I_x I_y$ and $I'^2_y = \alpha^2 I_y^2$. To calculate the corner measure we have to build the matrix

$$C(x, y) = \begin{pmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{pmatrix}$$

The modified matrix is given by

$$C'(x, y) = \alpha^2 \begin{pmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{pmatrix} = \alpha^2 C(x, y)$$

Plugging this into the Noble measure results in

$$c'_{Noble} = \alpha^2 c_{Noble}$$

Even though the actual corner measure changes, by selecting for instance the 100 strongest points, the same points will be obtained.

- b) This is not true for the Harris measure, since here the modified corner measure depends on α^2 , so choosing the 100 best points will not result in the same point set.

$$c'_{Harris} = \alpha^2 (\alpha^2 \det(C(x, y)) - k \cdot \text{trace}(C(x, y)))$$

Exercise 26 Normalized Cross Correlation (Exemplary Solution)

- a) The sample mean is defined as

$$\mu = \frac{1}{N} \sum_i x_i$$

and the sample variance is defined as

$$\sigma^2 = \frac{1}{N-1} \sum_i (x_i - \mu)^2$$

Hence:

$$\mu' = \frac{1}{N} \sum_i (\alpha x_i + \beta) = \frac{1}{N} (\alpha \sum_i x_i + N\beta) = \alpha \mu + \beta$$

and

$$\sigma'^2 = \frac{1}{N-1} \sum_i (\alpha x_i + \beta - (\alpha \mu + \beta))^2 = \frac{1}{N-1} \alpha^2 \sum_i (x_i - \mu)^2 = \alpha^2 \sigma^2$$

Hence

$$\sigma' = \alpha \sigma$$

- b) By substituting the terms obtained in a) into the definition of the NCC, you can see that the factors cancel out.
- c) When the standard deviation of one patch is zero the denominator is zero and hence the NCC is undefined. One extension to the NCC is to define that two patches with $\sigma = 0$ have $NCC = 1$.

Exercise 27 Image Stitching (Exemplary Solution)

- a) The projection matrix for the first image can be chosen as $\mathbf{P}_1 = \mathbf{K}_1[\mathbf{I}|\mathbf{0}]$ without loss of generality. The second projection matrix can then be chosen as $\mathbf{P}_2 = \mathbf{K}_2[\mathbf{R}|\mathbf{0}]$. It includes an arbitrary rotation and can also contain different internal camera parameters. This means that you can for instance also zoom on top of rotating the camera.

Let $\mathbf{X} = \begin{pmatrix} \mathbf{x} \\ w \end{pmatrix}$ be any world point. It's projection into the first image is given by

$$\begin{aligned} \mathbf{x}_1 &= \mathbf{K}_1 \mathbf{x} \\ \Rightarrow \mathbf{x} &= \mathbf{K}_1^{-1} \mathbf{x}_1 \end{aligned} \quad (1)$$

and it's projection into the second image is

$$\begin{aligned} \mathbf{x}_2 &= \mathbf{K}_2 \mathbf{R} \mathbf{x} \\ \Rightarrow \mathbf{x}_2 &= \mathbf{K}_2 \mathbf{R} \mathbf{K}_1^{-1} \mathbf{x}_1 \end{aligned} \quad (2)$$

We see that \mathbf{x}_1 and \mathbf{x}_2 are related by the homography $\mathbf{H} = \mathbf{K}_2 \mathbf{R} \mathbf{K}_1^{-1}$.

- b) Let's assume we have an arbitrary world point $\mathbf{X} = \begin{pmatrix} \mathbf{x} \\ w \end{pmatrix}$, which lies on a plane $\pi = \begin{pmatrix} \mathbf{n} \\ d \end{pmatrix}$. The projection matrix for the first image can be chosen as $\mathbf{P}_1 = \mathbf{K}_1[\mathbf{I}|\mathbf{0}]$ without loss of generality and the projection matrix for the second image as $\mathbf{P}_2 = \mathbf{K}_2[\mathbf{R}|\mathbf{t}]$. What we need to show now, is that the projection of \mathbf{X} into the first image at $\mathbf{x}_1 = \mathbf{P}_1 \mathbf{X}$ and into the second image at $\mathbf{x}_2 = \mathbf{P}_2 \mathbf{X}$ is related by a homography \mathbf{H} , such that $\mathbf{x}_2 = \mathbf{H} \mathbf{x}_1$. Using the fact that \mathbf{X} lies on π , we can write

$$\begin{aligned} \mathbf{X}^\top \pi &= 0 \\ \Rightarrow \mathbf{x}^\top \mathbf{n} + wd &= 0 \\ \Rightarrow w &= -\frac{\mathbf{x}^\top \mathbf{n}}{d} \end{aligned} \quad (3)$$

The projection of \mathbf{X} into the first image is given by

$$\begin{aligned} \mathbf{x}_1 &= \mathbf{P}_1 \mathbf{X} \\ &= \mathbf{K}_1 \mathbf{x} \\ \Rightarrow \mathbf{x} &= \mathbf{K}_1^{-1} \mathbf{x}_1 \end{aligned} \quad (4)$$

Using these equations we can write the projection of the point \mathbf{X} into the second image as:

$$\begin{aligned}
\mathbf{x}_2 &= \mathbf{P}_2 \mathbf{X} = \mathbf{K}_2 [\mathbf{R} | \mathbf{t}] \begin{pmatrix} \mathbf{x} \\ w \end{pmatrix} \\
&= [\mathbf{K}_2 \mathbf{R} | \mathbf{K}_2 \mathbf{t}] \begin{pmatrix} \mathbf{x} \\ w \end{pmatrix} \\
&= [\mathbf{K}_2 \mathbf{R} | \mathbf{K}_2 \mathbf{t}] \begin{pmatrix} \mathbf{x} \\ -\frac{\mathbf{x}^\top \mathbf{n}}{d} \end{pmatrix} \\
&= \mathbf{K}_2 \mathbf{R} \mathbf{x} - \mathbf{K}_2 \frac{\mathbf{t} \mathbf{x}^\top \mathbf{n}}{d} \\
&= \mathbf{K}_2 \mathbf{R} \mathbf{x} - \mathbf{K}_2 \frac{\mathbf{t} \mathbf{n}^\top \mathbf{x}}{d} \\
&= \left(\mathbf{K}_2 \mathbf{R} - \mathbf{K}_2 \frac{\mathbf{t} \mathbf{n}^\top}{d} \right) \mathbf{x} \\
&= \mathbf{K}_2 \left(\mathbf{R} - \frac{\mathbf{t} \mathbf{n}^\top}{d} \right) \mathbf{x} \\
&= \mathbf{K}_2 \left(\mathbf{R} - \frac{\mathbf{t} \mathbf{n}^\top}{d} \right) \mathbf{K}_1^{-1} \mathbf{x}_1
\end{aligned} \tag{5}$$