



A Unified Approach Combining Photometric and Geometric Information for Pose Estimation

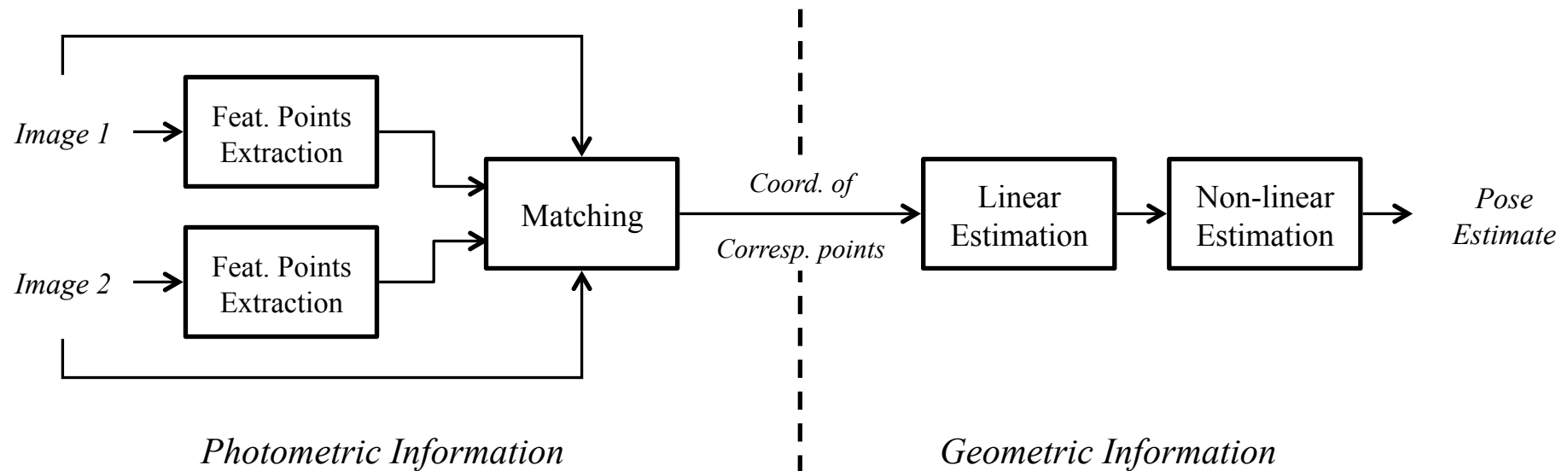
Pierre Georgel



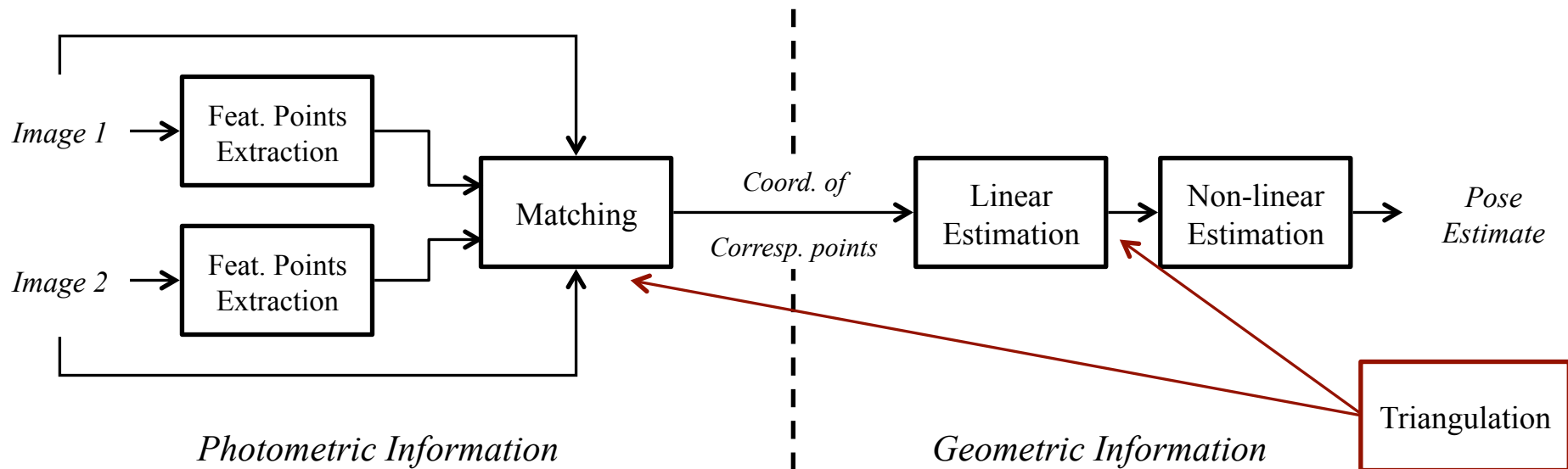
Agenda

- Reminder
 - Triangulation
 - Pose estimation
 - Bundle adjustment
- Hybrid registration
 - New cost function
 - Pit fall to avoid

What you know



What you know

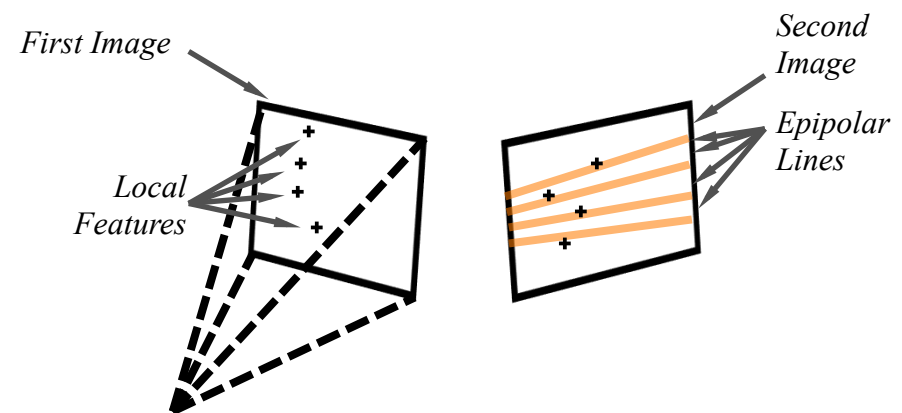


Epipolar Geometry *Fast Forward*

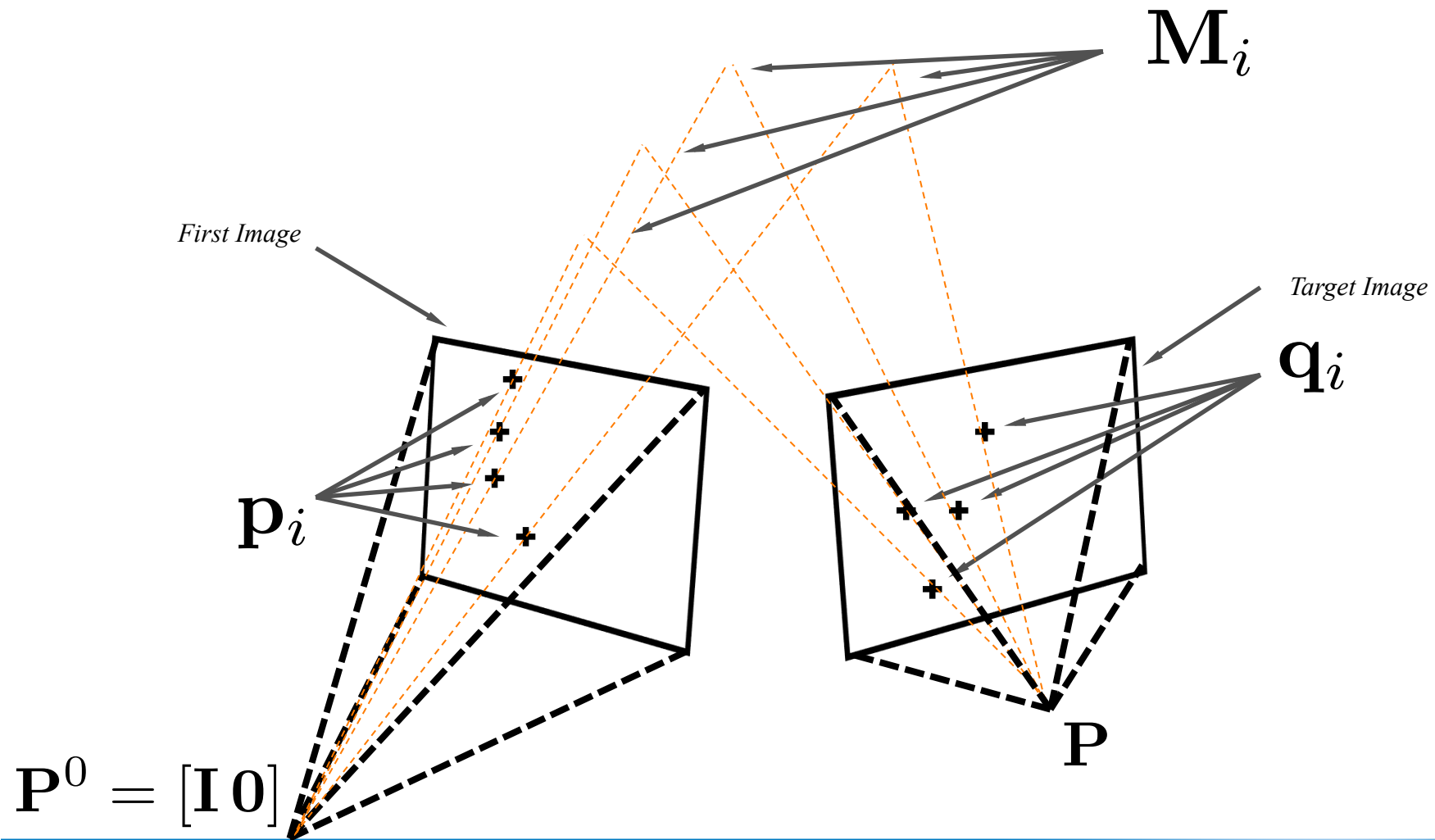
- Compute fundamental matrix using keypoints $\langle \mathbf{p}_i, \mathbf{q}_i \rangle$
- This is done by minimize a cost function, for example the epipolar constraint

$$\mathbf{q}_i^\top \mathbf{F} \mathbf{p}_i = 0$$

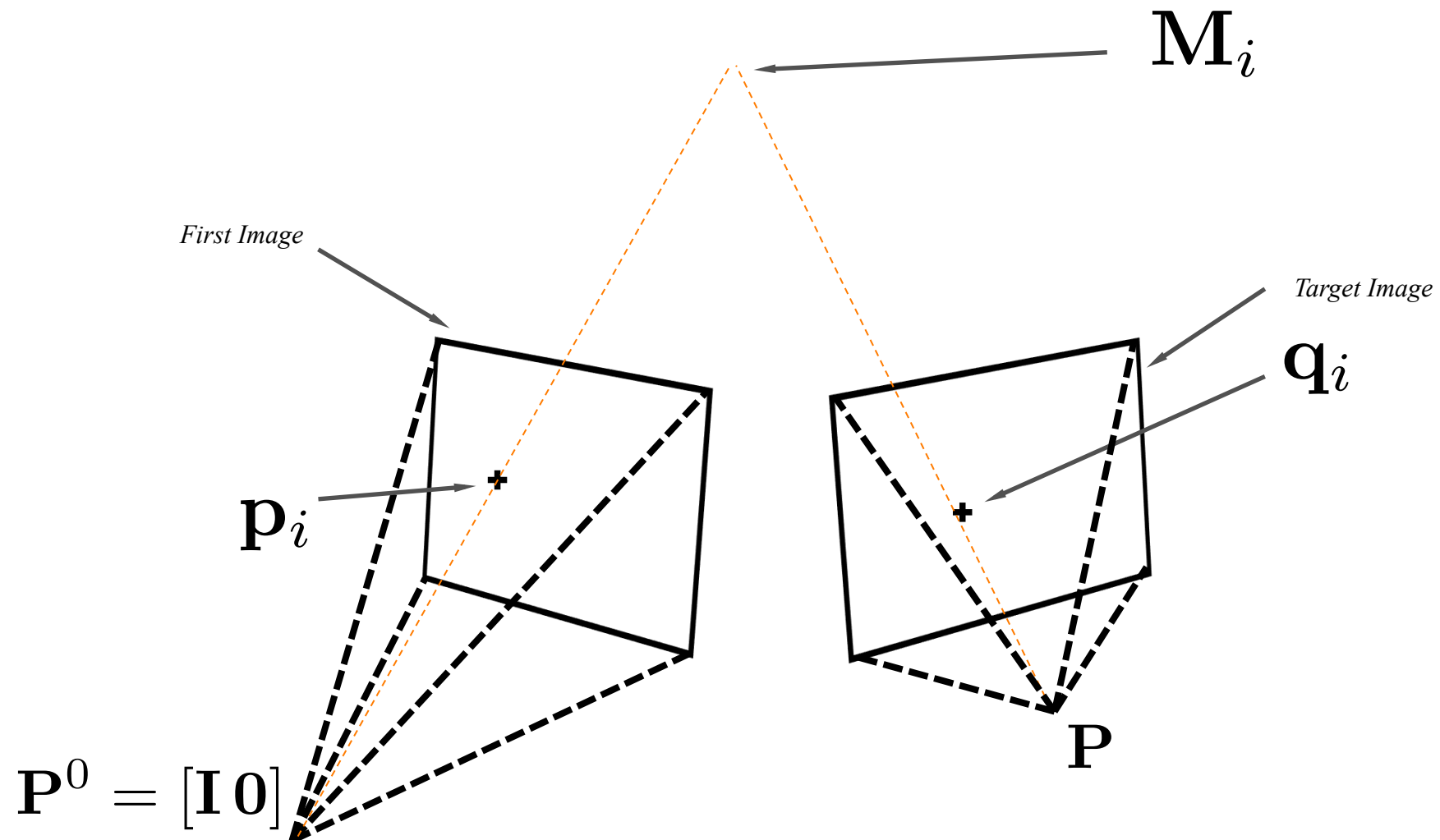
- Derive a canonical pose \mathbf{P}



Triangulation



Triangulation



Triangulation

$$\begin{cases} \mathcal{M} & \sim & \mathbf{p} \\ \mathbf{P}\mathcal{M} & \sim & \mathbf{q} \end{cases}$$

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Triangulation

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Triangulation

$$\left\{ \begin{array}{l}
 \mathcal{M} \sim \mathbf{p} \\
 \mathbf{P}\mathcal{M} \sim \mathbf{q} \\
 \mathcal{M} = z\mathbf{p} \\
 \mathbf{P}\mathcal{M} = z'\mathbf{q} \quad \text{with } \mathbf{P}^{0\top}\mathcal{M} = z \text{ and } \mathbf{P}_3\mathcal{M} = z' \\
 \mathbf{P}_1^{0\top}\mathcal{M} - x\mathbf{P}_3^{0\top}\mathcal{M} = 0 \\
 \mathbf{P}_2^{0\top}\mathcal{M} - y\mathbf{P}_3^{0\top}\mathcal{M} = 0 \\
 \mathbf{P}_1^\top\mathcal{M} - x'\mathbf{P}_3^\top\mathcal{M} = 0 \\
 \mathbf{P}_2^\top\mathcal{M} - y'\mathbf{P}_3^\top\mathcal{M} = 0
 \end{array} \right.$$

Triangulation

$$\left\{ \begin{array}{l} (\mathbf{P}_1^{0\top} - x\mathbf{P}_3^{0\top})\mathcal{M} = 0 \\ (\mathbf{P}_2^{0\top} - y\mathbf{P}_3^{0\top})\mathcal{M} = 0 \\ (\mathbf{P}_1^{\top} - x'\mathbf{P}_3^{\top})\mathcal{M} = 0 \\ (\mathbf{P}_2^{\top} - y'\mathbf{P}_3^{\top})\mathcal{M} = 0 \end{array} \right.$$

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- We have a linear system express with \mathbf{M} as unknown

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- We have a linear system express with \mathbf{M} as unknown
- You can extend this two n views

Triangulation

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- We have a linear system express with \mathbf{M} as unknown
 - You can extend this two n views
 - But what it ultimately does is an intersection in 3D which has little to do with observation
-

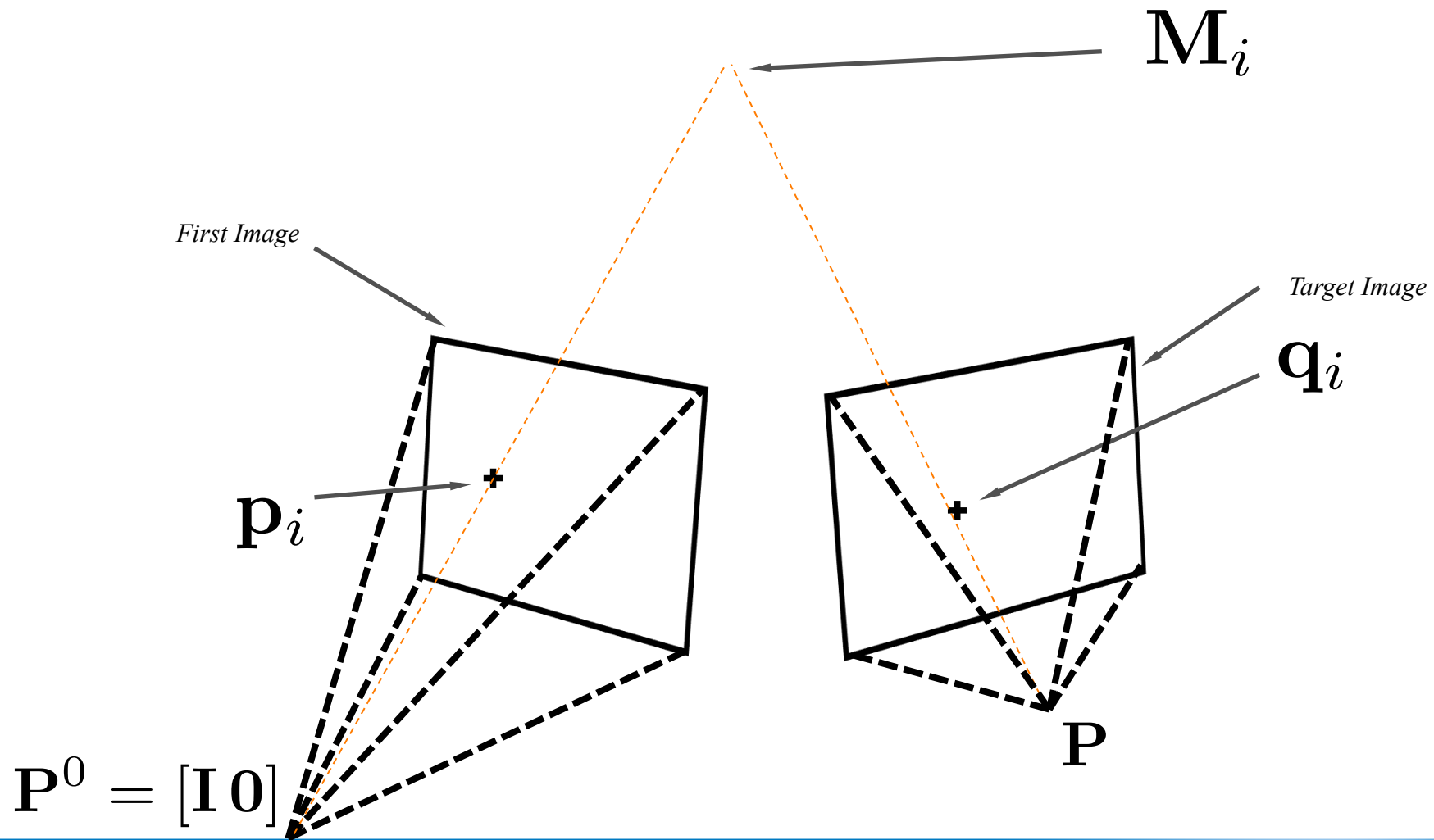
Triangulation

Non-linear estimation

- You want to optimize the re-projection error

$$\arg \min_{\mathcal{M}} \sum_i \left\| \mathbf{w} (\mathbf{P}^i \mathcal{M}) - \mathbf{p}^i \right\|^2$$

Triangulation



Triangulation

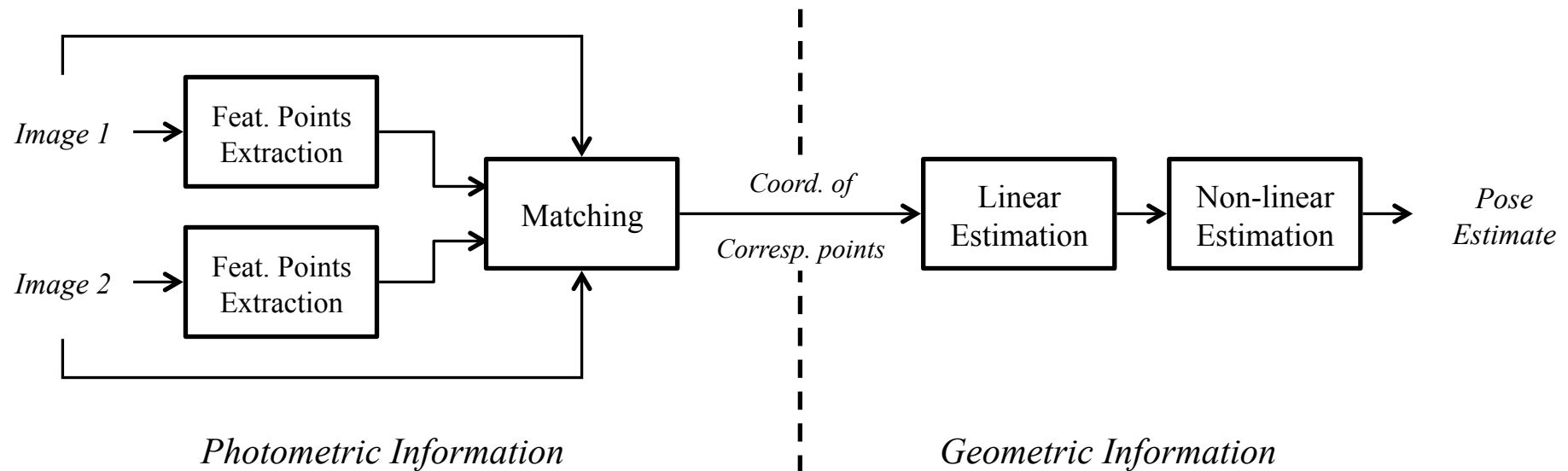
Non-linear estimation

- You want to optimize the re-projection error

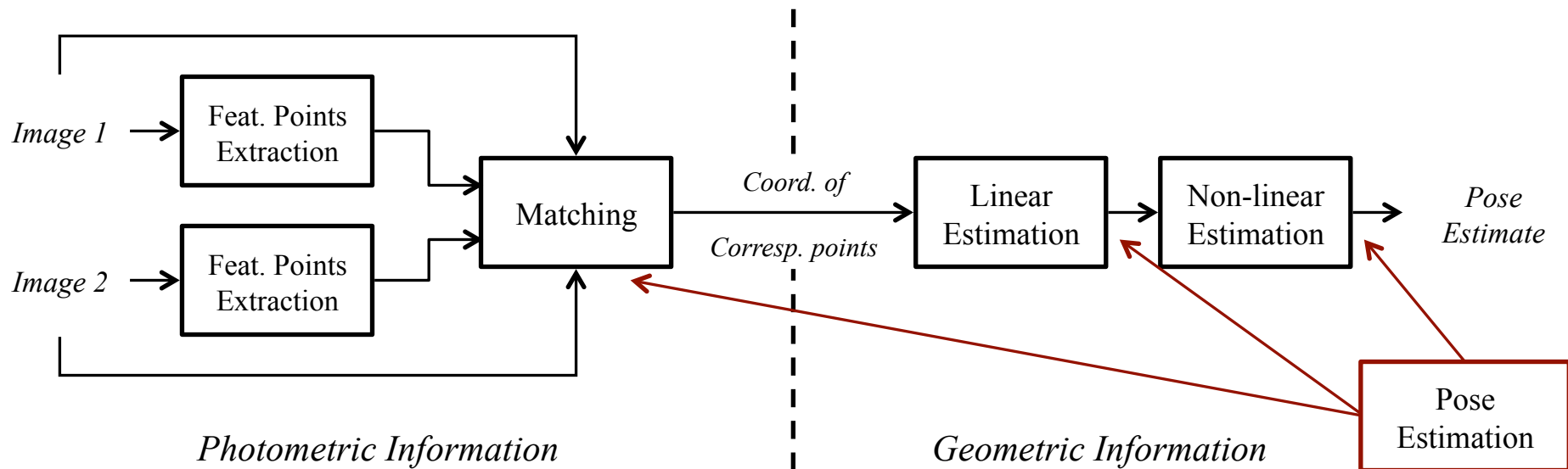
$$\arg \min_{\mathcal{M}} \sum_i \left\| \mathbf{w} (\mathbf{P}^i \mathcal{M}) - \mathbf{p}^i \right\|^2$$

- This is a least square problem

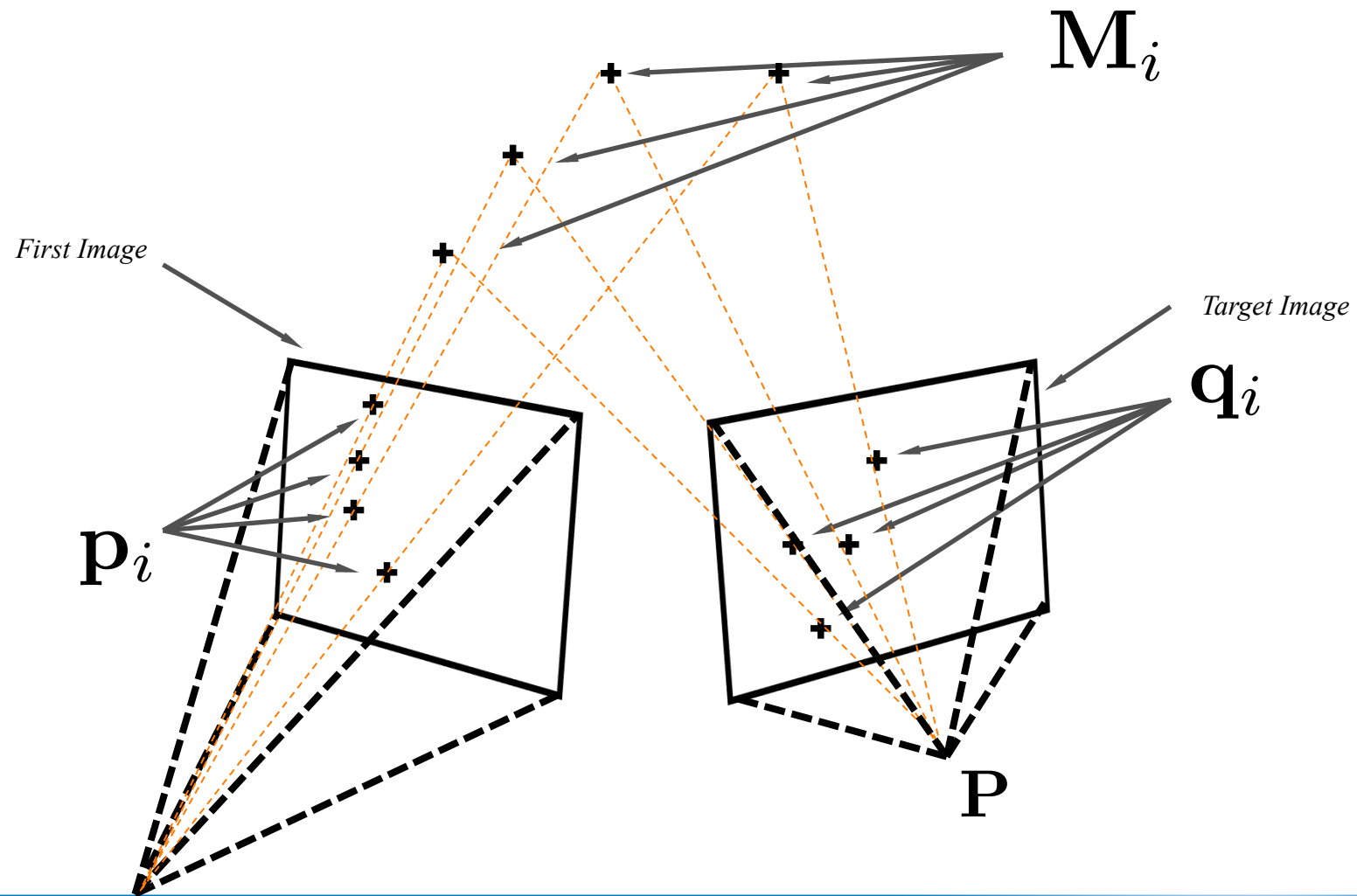
What you know



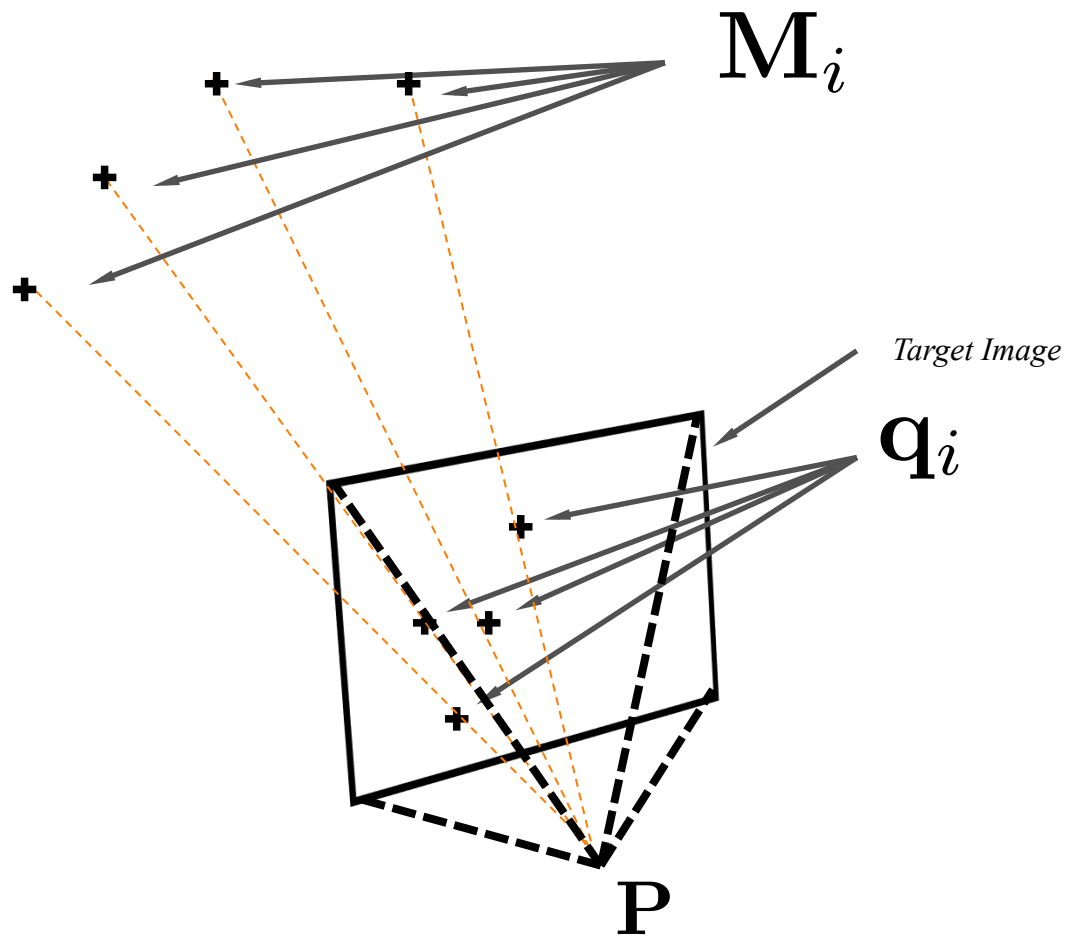
What you know



Pose Estimation

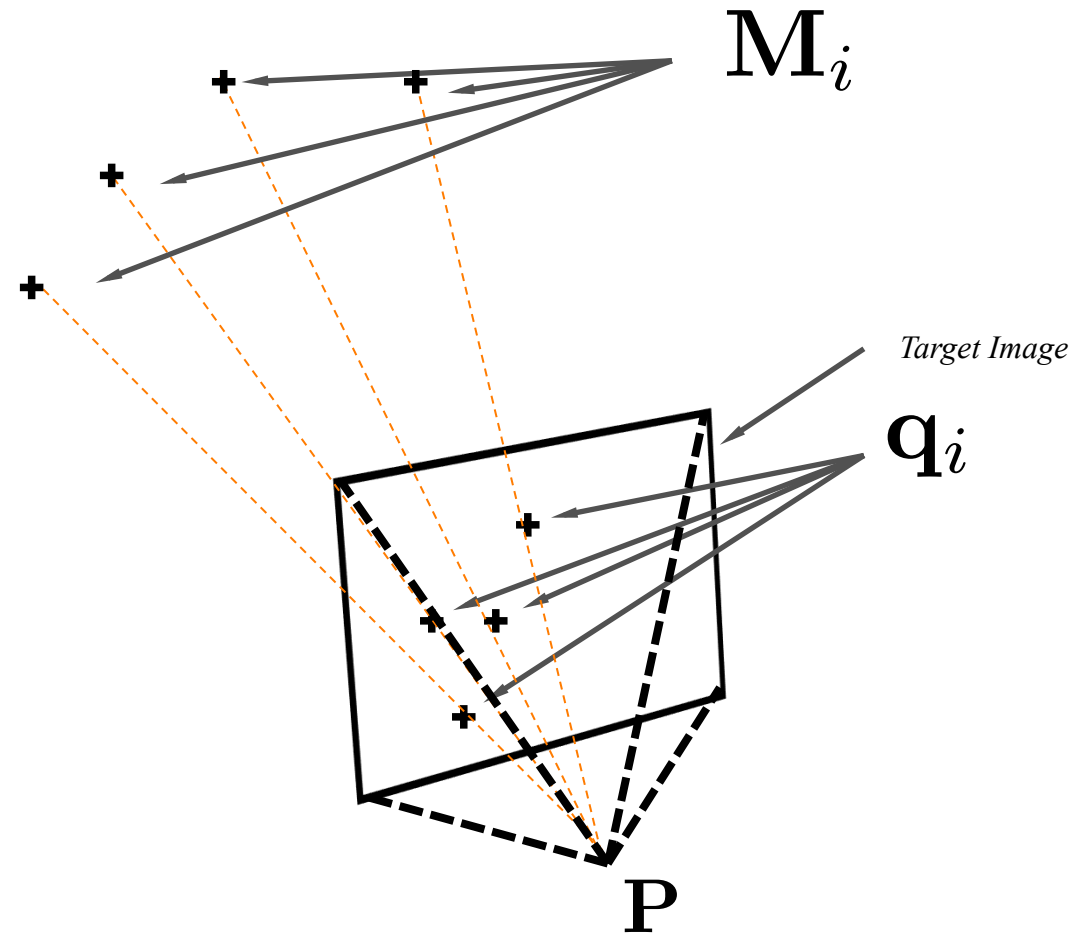


Pose Estimation



Pose Estimation

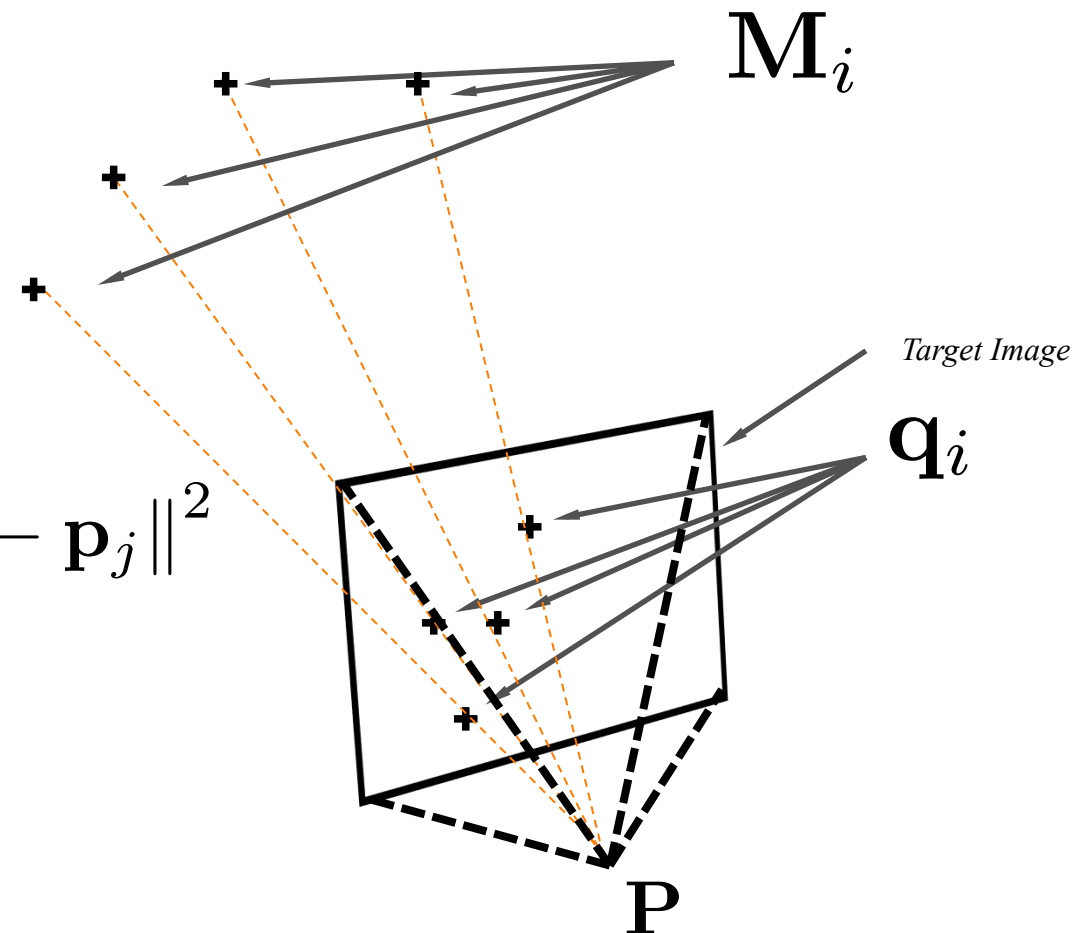
- Using the same idea as for triangulation you can build a linear system



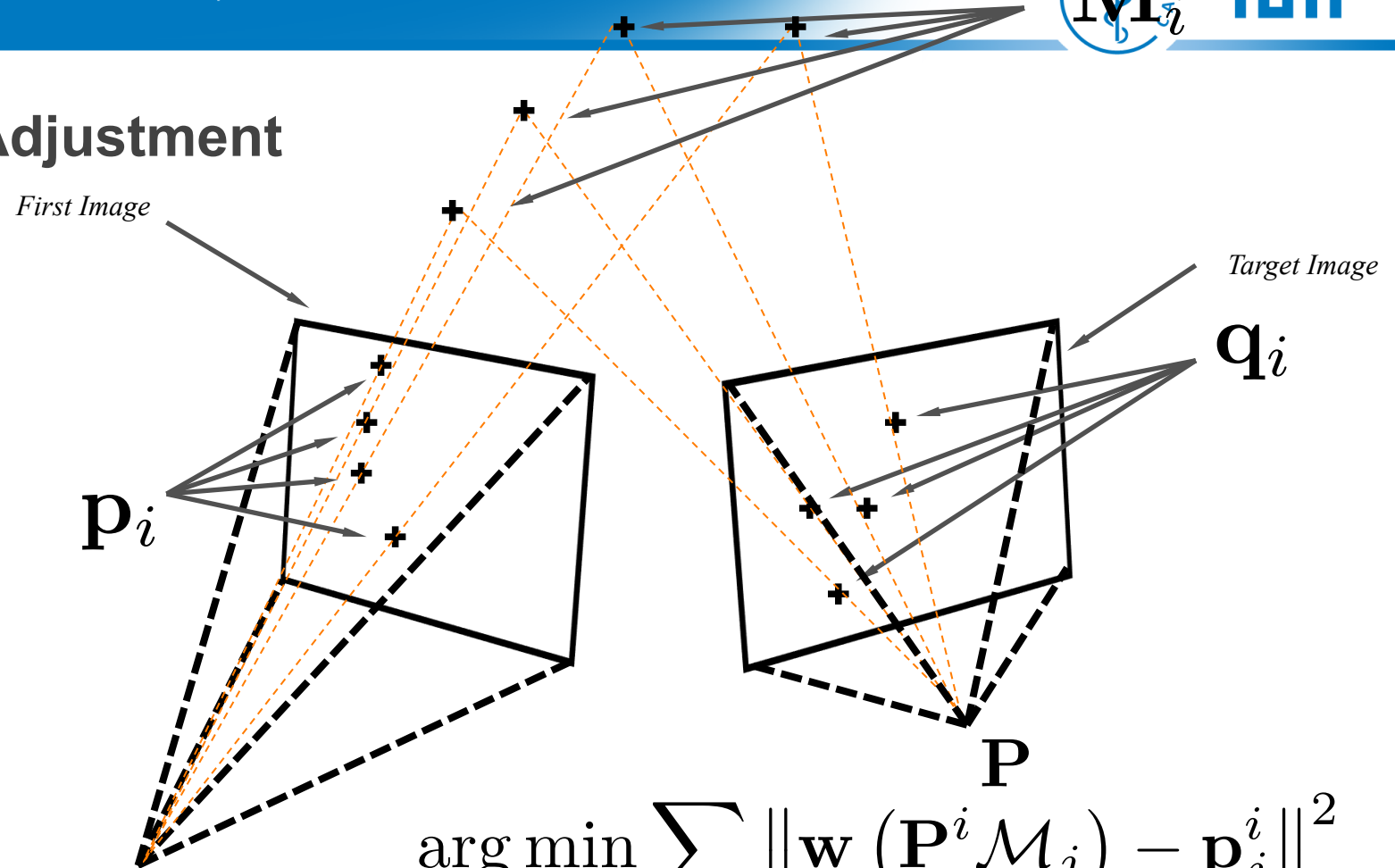
Pose Estimation

- Using the same idea as for triangulation you can build a linear system
- Least square related cost function

$$\arg \min_{\mathcal{P}} \sum_j \| \mathbf{w} (\mathbf{P} \mathcal{M}_j) - \mathbf{p}_j \|^2$$



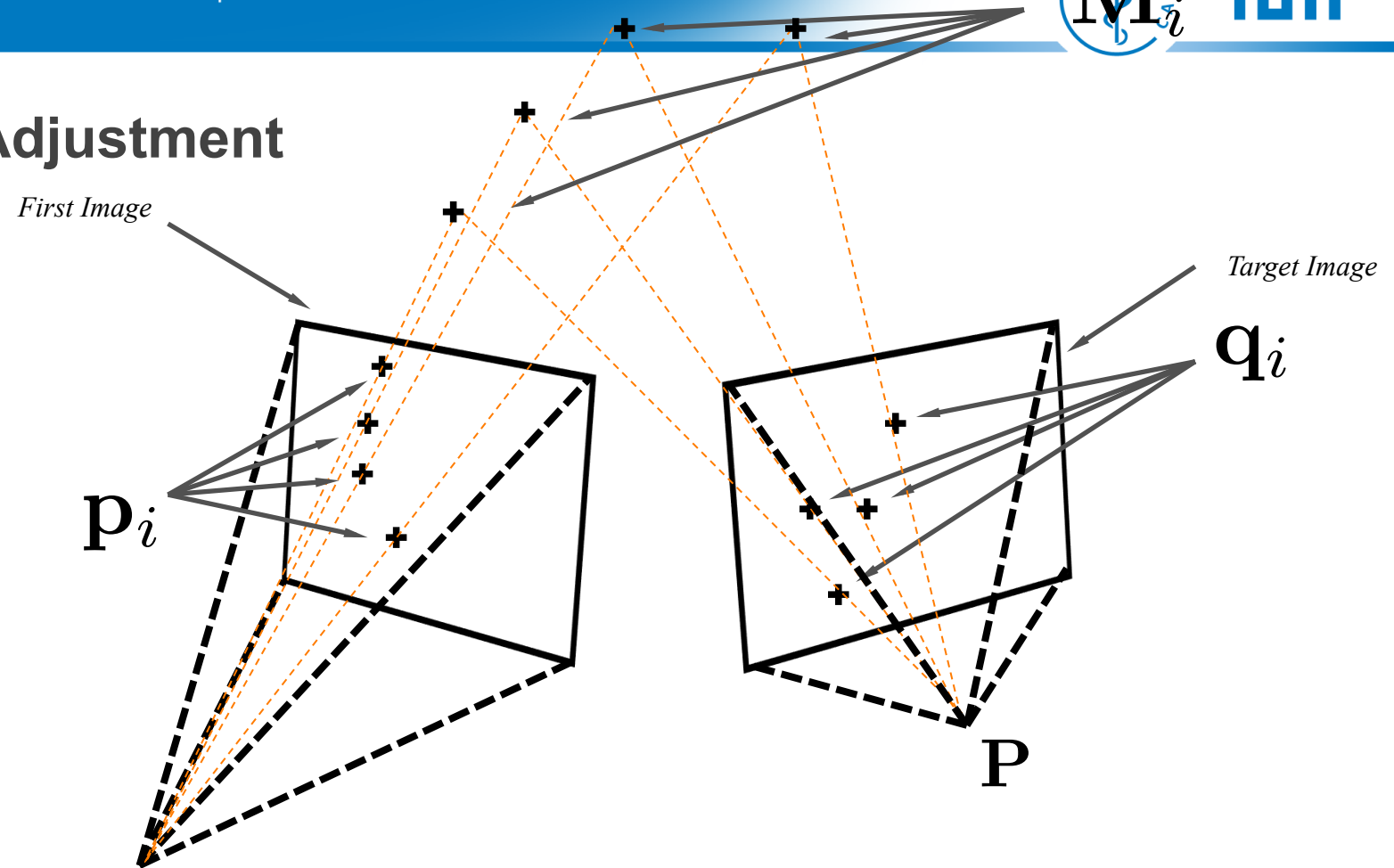
Bundle Adjustment



$$\arg \min_{\mathcal{M}_j} \sum_{i,j} \|\mathbf{w} (\mathbf{P}^i \mathcal{M}_j) - \mathbf{p}_j^i\|^2$$

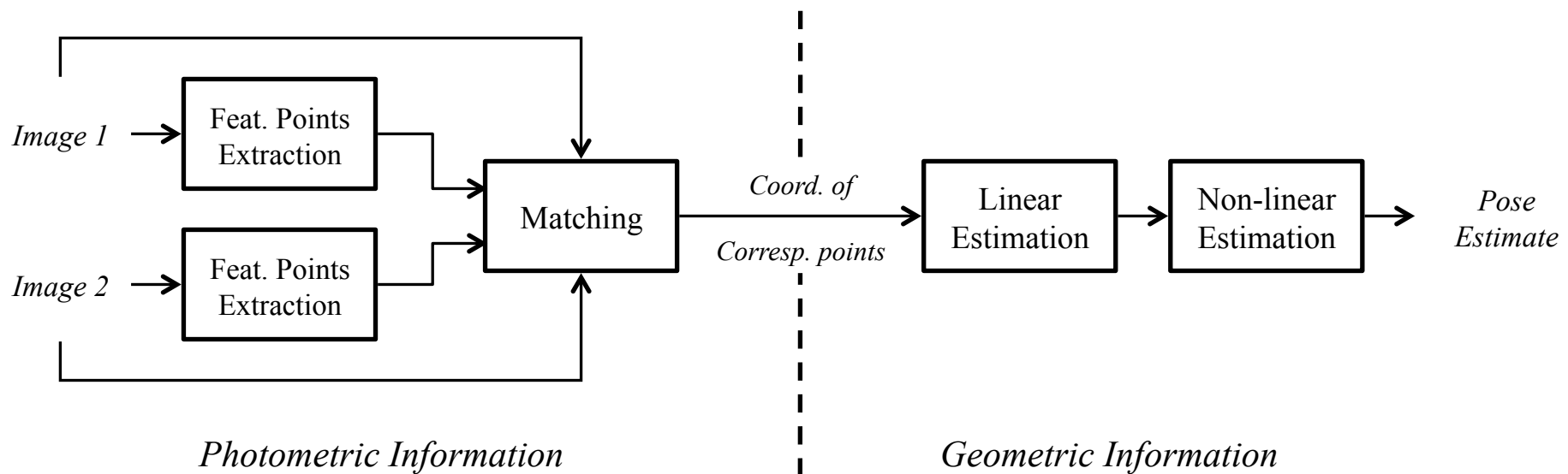
$$\arg \min_{\mathcal{P}} \sum_j \|\mathbf{w} (\mathbf{P} \mathcal{M}_j) - \mathbf{p}_j\|^2$$

Bundle Adjustment



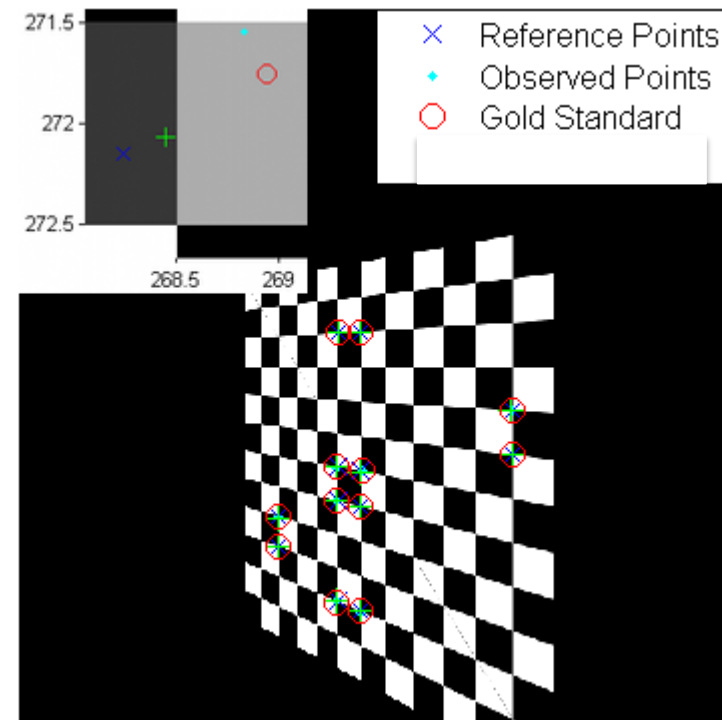
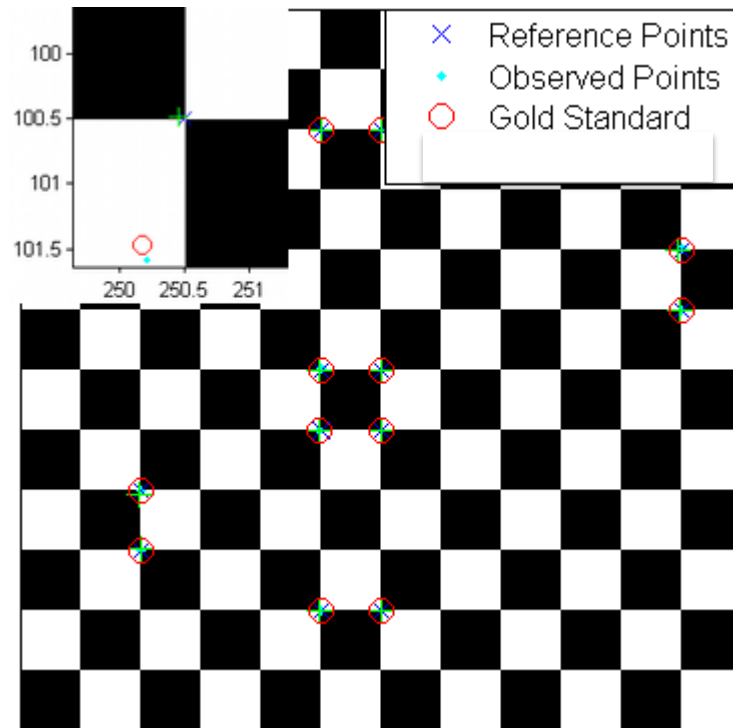
$$\arg \min_{\mathbf{P}^i, \mathcal{M}_j} \sum_{i,j} \left\| \mathbf{w} \left(\mathbf{P}^i \mathcal{M}_j \right) - \mathbf{p}_j^i \right\|^2$$

What you know

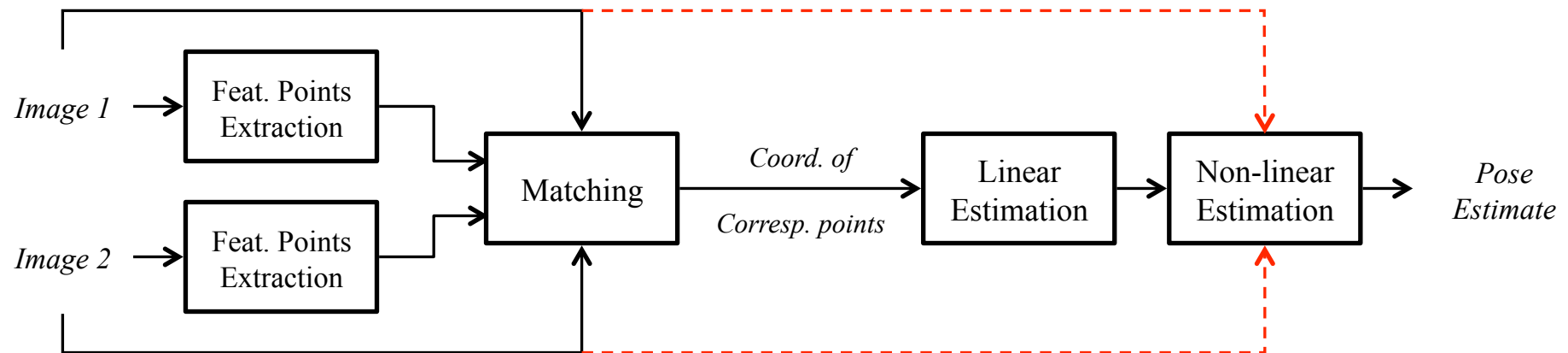


$$\arg \min_{\mathcal{X}_i, \mathbf{T}} \sum_i d(\mathbf{p}_i, \mathbf{K}\mathbf{w}(\mathcal{X}_i))^2 + d(\mathbf{p}'_i, \mathbf{K}'\mathbf{w}(\mathbf{T}\mathcal{X}_i))^2$$

Is Geometric Only a Good Idea?



What can we do more?



Photometric Information and Geometric information in a unified framework

Modification of the cost function

$$\sum_{j \in \Omega_i} (\mathcal{I}_1(\mathbf{K}\mathbf{w}(\mathcal{Y}_{ij})) - \mathcal{I}_2(\mathbf{K}'\mathbf{w}(\mathbf{T}\mathcal{Y}_{ij})))^2$$

- How can we define the neighborhood of the 2D points in the two images?
- At which stage the photometric term should be used?
- How should the geometric and the photometric terms be weighted?

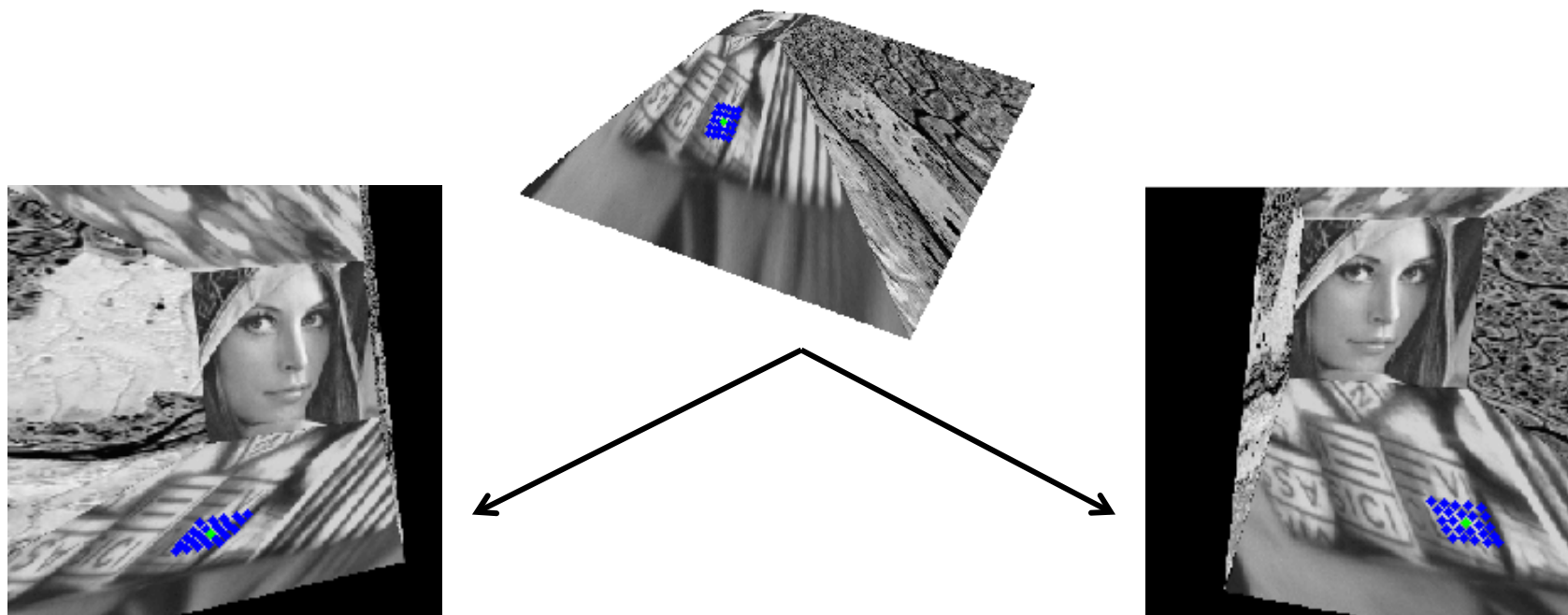


Figure 3: The Neighbors \mathcal{Y}_{ij} (in blue) defined in 3D around triangulated point \mathcal{X}_i (in green), they are projected in the image to create Patch_i and Patch'_i as shown in left and right pictures.

When to activate the Intensity based cost function?

#Points/Noise	0.01	0.1	1.0	2.0
8	0.3619	3.9054	96.3384	179.4612
10	0.2066	1.9684	17.0605	24.6839
25	0.1002	1.0511	10.4050	18.5587
50	0.0660	0.6916	6.0568	14.0087

Table 1: Evolution of the mean residual error (in pixels) over 200 runs after applying the 8-points algorithm and the optimal triangulation, with respect to the number of points and the Gaussian noise.

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Table 1: Evolution of the mean residual error (in pixels) over 200 runs after applying the 8-points algorithm and the optimal triangulation, with respect to the number of points and the Gaussian noise.

$$\delta_i = \begin{cases} 1 & \text{if} \\ 0 & \text{otherwise} \end{cases} \begin{cases} d(\mathbf{p}_i, \mathbf{K}\mathbf{w}(\mathcal{R}_i)) < \tau_1 \text{ and } d(\mathbf{p}'_i, \mathbf{K}'\mathbf{w}(\mathbf{T}\mathcal{R}_i)) < \tau_1 \\ \text{NCC}(\text{Patch}_i, \text{Patch}'_i) < \tau_2 \end{cases}$$

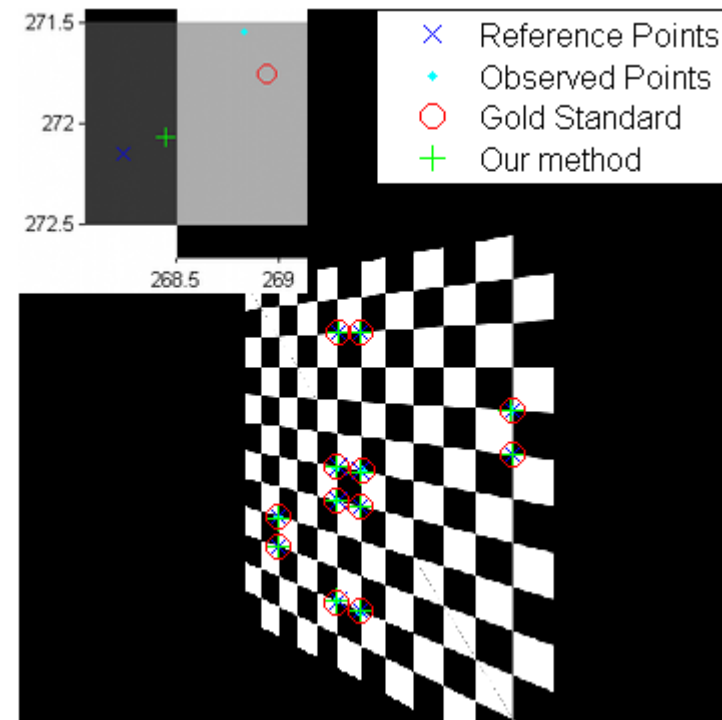
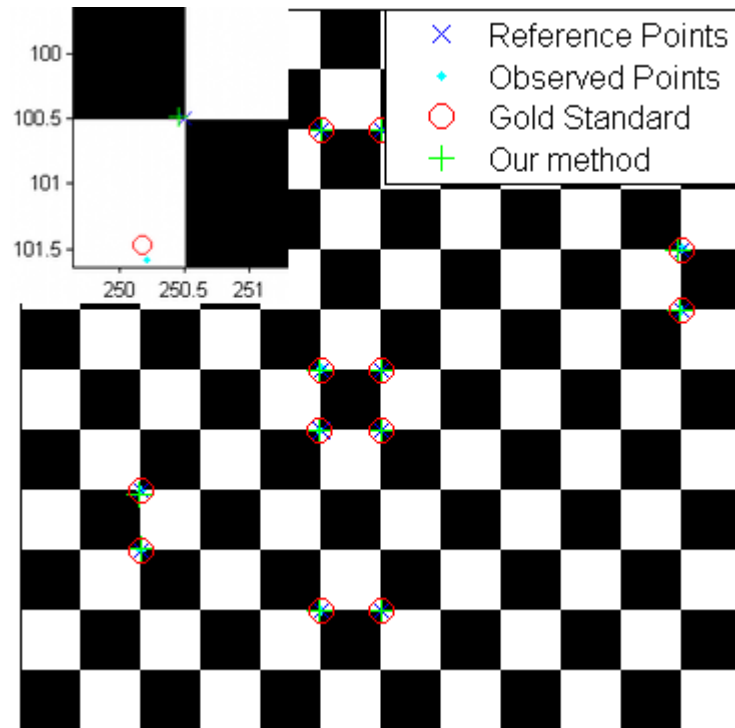
Unified Cost function

$$\arg \min_{\mathcal{X}_i, \mathbf{T}} \alpha_g \mathcal{D}_g(\mathcal{X}_i, \mathbf{T}) + \alpha_p \mathcal{D}_p(\mathcal{X}_i, \mathbf{T})$$

$$\mathcal{D}_g = \sum_i d(\mathbf{p}_i, \mathbf{K}\mathbf{w}(\mathcal{X}_i))^2 + d(\mathbf{p}'_i, \mathbf{K}'\mathbf{w}(\mathbf{T}\mathcal{X}_i))^2$$

$$\mathcal{D}_p = \sum_i \delta_i \sum_j (\mathcal{I}_1(\mathbf{K}\mathbf{w}(\mathcal{Y}_{ij})) - \mathcal{I}_2(\mathbf{K}'\mathbf{w}(\mathbf{T}\mathcal{Y}_{ij})))^2$$

Qualitative Results



Quantitative Results

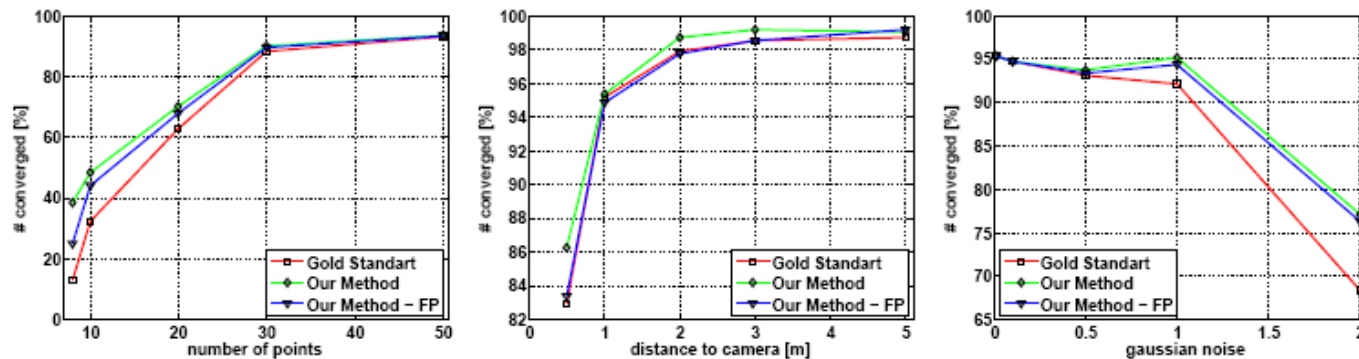


Figure 4: Convergence rate in percent; the first graph shows results with an increasing number of points and a Gaussian noise of 0.5; the second graph shows the effect of an increasing distance from the scene with 50 points and a Gaussian noise of 0.5; the last graph shows the results with 50 points and varying Gaussian noise; it shows that the additional intensity information enables our approach to be more robust even with a fronto-parallel assumption (FP).



Conclusion

- Lesson learned
 - Look at what the linear cost function try to solve
 - Think about what you really want to minimize

 - Using both information we achieve
 - Better convergence rate
 - Better precision
 - Seems to work for calibration



Merry Christmas!

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