Rectification and Disparity

Nassir Navab

Slides prepared by Christian Unger
What is Stereo Vision?

Introduction

• A technique aimed at inferring dense depth measurements efficiently using two cameras.
• Wide and old research area in computer vision.
• Contents of the talks:
  – Radial Undistortion: Compensate effects of radial lens distortion.
  – Rectification: Transforming the epipolar geometry into a canonical form.
  – Disparity: Definition and relation to depth.
  – Binocular Stereo Matching: The process of establishing dense correspondences between two images.
  – Triangulation: Computing a 3D reconstruction.
Stereo-Rigs and Applications.

NASA
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Exploration

A Stereo Vision System is usually composed of these processing steps:
1. Radial Undistortion
2. Rectification
3. Stereo Matching
4. Triangulation

- Radial Undistortion
- Rectification
- Stereo Matching
- Triangulation

Compensate for the effects of radial lens distortion.

Use calibration patterns to make straight lines straight.
Radial Lens Distortion: Examples.
Radial Lens Distortion: Mathematical Background.

We use a simplified version of Brown’s model (barrel distortion) to formulate the undistortion:

- The raw-camera-image contains distorted image points:
  \[ x_D = (x_D, y_D) \]

- We seek the undistorted image points
  \[ x_U = (x_U, y_U) \]

- Brown’s model is given as:
  \[ x_U = x_D + L(r) \cdot (x_D - x_C) \]

- With
  \[ L(r) = K_1 r^2 + K_2 r^4 + \ldots \]
  \[ r = \sqrt{(x_D - x_C)^2 + (y_D - y_C)^2} \]
Radial Lens Undistortion: Approach.

Offline phase:
• Estimate the distortion parameters using 2D-3D correspondences (e.g. using a checker-board).

Online phase:
• Warp images using a reverse distortion ("backward warping": for every undistorted pixel we have to compute the distorted location).
• Due to the non-linearity of the distortion-function, this must be done numerically.
• In practice, usually a lookup-table is computed offline.

Align the image planes (move the epipoles to infinity and match up epipolar lines).
Reminder: Depth from Corresponding Points.

With two (or more) cameras we can infer depth from correspondences using **triangulation**.
Reminder: Epipolar Geometry.

Since correspondences are constrained by epipolar geometry, only a one-dimensional search domain has to be considered:

\[ l' = Fx \]
Rectification: Stereo in Canonical Form.

In general: epipolar lines may be slanted.
   => Computation of the epipolar line for every image point is costly!

It is more desirable to have parallel, horizontal epipolar lines.
   => Simple search along scanlines!
Rectification: Stereo in Canonical Form.
Rectification: Stereo in Canonical Form.

Rotate the cameras virtually, so that the two image-planes become co-planar.
Rectification: Stereo in Canonical Form.
Rectification: An Interpretation.

In General:
Rectification = Put the epipoles to a predefined position
= Align the image planes (rotation, focal length)

Standard approach:
• Make epipolar lines run parallel to the x-axis. For that,
• move the epipole to infinity and
• match up epipolar lines between views.

By this convention, points will only move in the x-direction – there will be no movement in y-direction!
Rectification: Minimize Image Distortion.

Let \( H' \) be a projective transformation that sends the epipole \( e' \) to infinity. Then, the transformation \( H \) for the second camera might be chosen so as to minimize the sum-of-squared distances:

\[
\sum_i d(Hx_i, H'x_i')^2
\]

Furthermore, \( H \) might be constrained to match up corresponding epipolar lines:
Rectification: Algorithm Outline.

1. Find initial correspondences
2. Compute the fundamental matrix
3. Compute a projective transformation $H'$ that maps the epipole $e'$ to infinity $(1, 0, 0)^T$
4. Find the matching projective transformation $H$ that minimizes
   \[
   \sum_i d(Hx_i, H'x'_i)^2
   \]
5. Warp the first image according to $H$ and the second image according to $H'$
Rectification: Another Interpretation.

In practice the presented algorithm (Hartley-Zisserman) works, but may lead to undesirable results (e.g. „black borders“).

Argumentation via the DOF of a homography:
• Moving the epipole to infinity constrains only 2 DOF.
• Some clarifications and repetitions:
  – Epipolar lines intersect at the epipole.
  – The image of the baseline in rectified images is parallel to the x-axis.
• Now, we use four line-to-line correspondences:
  – We use 2 epipolar lines to constrain top and bottom image borders
  – We use another 2 lines being orthogonal to the baseline to constrain left and right image borders
  – These four lines define a rectangle => ensure that the rectangle is completely within the FOV.
Rectification: Another Interpretation.

=>

- Make **top** and **bottom** lines horizontal, to move the epipole to infinity.
- Map **left** and **right** lines to the left and right image border to avoid “black regions”).

Given lines in the left image, how to obtain the lines in the right image:

- Use corresponding epipolar lines (for **top** and **bottom**).
- Use points at infinity to ensure a proper disparity offset and scale for (**left** and **right**).
Rectification: Another Interpretation.

=>

- Make top and bottom lines horizontal, to move the epipole to infinity.
- Map left and right lines to the left and right image border to avoid “black regions”).

Given lines in the left image, how to obtain the lines in the right image:
- Use corresponding epipolar lines (for top and bottom).
- Use points at infinity to ensure a proper disparity offset and scale for (left and right).
Disparity: Definition.

Assume a calibrated, rectified stereo setup.

**Definition (Disparity)**

*The disparity measures the displacement of a point between the two images.*

- **Left Cam**
- **Right Cam**

Close points: large displacement
Far points: small displacement
Disparity: Relationship to Depth.

Left Cam \( x_L \)  

Right Cam \( x_R \)

Depth \( Z \)  

Baseline \( b \)  

Focal length \( f \)
Disparity: Relationship to Depth.

Similar Triangles!
Disparity: Relationship to Depth.

We get:

\[
\frac{b}{Z} = \frac{(b + x_R) - x_L}{Z - f} \Rightarrow d = x_L - x_R = \frac{f \cdot b}{Z}
\]
Disparity and the Baseline.

\[ d = \frac{b \cdot f}{Z} \]

Using this relationship we can draw important conclusions:

- Disparity values are inverse proportional to the depth of a point \( Z \):
  - Far points have low disparity (for example the horizon has disparity of zero).
  - Close points have a high disparity.

- The disparity is proportional to the baseline \( b \)
  - The larger the baseline, the higher the disparity.

- The disparity resolution scales linearly with imager resolution:
  - High resolutions allow accurate disparity measurements.
Disparity Maps: Dense Correspondences.

Disparity for every pixel in the image => **Disparity Map**
Typically encoded using intensities: “close points are bright – far ones are dark“
The Horopter.
Disparity-Planes.

Image of S. Mattoccia.

**Stereo Matching**
Compute a disparity for every pixel of the image.

Next Topic!

**Triangulation**
Compute the 3D position for every disparity.

Next but one Topic!