

Exercises in 3D Computer Vision I

Exercise 1 **Recap: Geometry of Points, Lines and Planes**

This exercise aims to refresh your knowledge about the representation of points, lines and planes in homogeneous coordinates. These rules will be used throughout the course.

- a) What is the homogeneous representation of points and lines in 2D?
- b) What is the homogeneous representation of points and planes in 3D?
- c) How can you determine the line \mathbf{l} through two points $\mathbf{x}_1, \mathbf{x}_2$ in homogeneous representation?
- d) How can you determine the intersection point \mathbf{x} of two lines $\mathbf{l}_1, \mathbf{l}_2$?
- e) How can you determine if a point lies on a line or on a plane?

Exercise 2 **Recap: Transformations**

In this exercise, we quickly recapitulate the basics of transformations in 2D and 3D. Consider the following types of transformations for the following questions: *Euclidean, Similarity, Affinity, Projectivity*.

- a) For each of the transformations above, give a matrix representation in **2D** and specify the number of degrees of freedom per transformation.
- b) Mark the right invariance properties for the transformations in **2D**.

	Euclidean	Similarity	Affinity	Projectivity
Area				
Angle				
Ratio of Lengths				
Parallelism				
Line at Infinity				
Intersection Points				
Tangency				

- c) For each of the transformations above, give a matrix representation in **3D** and specify the number of degrees of freedom per transformation.

Exercise 3 **Recap: Homographies and Geometric Error Models**

Homographies are projective 2D transformations that arise in perspective images, for instance in the following situations:

- Given a world plane and a (perspective) image of the plane, a homography relates points on the world plane $\{\bar{\mathbf{x}}_i\}$ to their corresponding points $\{\mathbf{x}_i\}$ in the image.

- Given two (perspective) images of a world plane, a homography relates points $\{\mathbf{x}_i \leftrightarrow \mathbf{x}'_i\}$ between the two images that correspond to the same points $\{\bar{\mathbf{x}}_i\}$ on the world plane.
- Given two (perspective) images of an arbitrary world scene that have been obtained by only rotating the camera about its center, a homography relates points $\{\mathbf{x}_i \leftrightarrow \mathbf{x}'_i\}$ between the two images that correspond to the same points $\{\bar{\mathbf{x}}_i\}$ in the world scene.

We would like to find a homography \mathbf{H} from point correspondences $\{\mathbf{x}_i \leftrightarrow \mathbf{x}'_i\}$, such that $\mathbf{x}'_i \propto \mathbf{H}\mathbf{x}_i$. In the beginning we first have to give an initial solution which can be computed by the DLT:

This exercise aims to quickly repeat the basics of the Direct Linear Transform algorithm (DLT) for homography computation. It is very similar to a linear algorithm for computation of the fundamental matrix which we will discuss in another Exercise.

- Outline the main steps of the DLT algorithm for homography computation. Assume you are given point correspondences $\{\mathbf{x}_i \leftrightarrow \mathbf{x}'_i\}_{i=1\dots N}$, where $\mathbf{x} = (x, y, w)^T$. How many point correspondences are required to compute a homography \mathbf{H} such that $\mathbf{x}' = \mathbf{H}\mathbf{x}$?
- In practice, the DLT as described above often does not give satisfactory results. Outline why normalization has to be applied to the point correspondences and how you would perform normalization and denormalization.

Once an initial solution is computed, one needs to refine it. This is because the point correspondences are measured points and not exact. For refining, we need an appropriate error model. Assume that the Euclidean distance between two points \mathbf{x}, \mathbf{y} is $d(\mathbf{x}, \mathbf{y})$.

- Which general types of error can you think of concerning homography estimation?
- The formula for the *simple transfer error* is $\sum_i d(\mathbf{x}'_i, \mathbf{H}\mathbf{x}_i)^2$. Give an intuitive explanation for this error model, e.g. using a drawing.
- What are the parameters used to minimize the simple transfer error?
- Give an intuitive explanation for the *symmetric transfer error* and the correct formula.
- What are the parameters used to minimize the symmetric transfer error?
- Give an intuitive explanation for the *reprojection error* and the correct formula, assuming the estimated correspondences are $\{\hat{\mathbf{x}}_i \leftrightarrow \hat{\mathbf{x}}'_i\}$.
- What are the parameters used to minimize the reprojection error?
- What are the advantages and drawbacks of the different error models?

Exercise 4 (H) Image Transformations in MATLAB

This exercise aims to familiarize yourself with image processing using MATLAB. You have learned how to apply 2D transformations, such as similarities, affinities or projectivities to single points. Now, we would like to apply these transformations to an entire image.

- Think of what problems occur when dealing with rastered image coordinates. How can you deal with these problems?
- Download the supplementary material from the lecture website. You are provided stubs for MATLAB functions `transform_image_forward` and `transform_image_backward`. Familiarize yourself with the ideas of forward and backward image warping.

- c) Implement forward and backward image warping by filling the missing code in the two respective function stubs. For the interpolation step in backward warping, you can use the provided function `bilinear_interpolation`.
- d) Write a MATLAB function `transform_image` that takes three arguments: an image, a 2D transformation matrix (homogeneous), and one of the strings `'forward'` or `'backward'`. The function is supposed to return the image transformed in the specified way.
- e) Create a MATLAB script that loads an image of your choice and applies one or more 2D transformations of your choice to it (e.g. rotation, anisotropic scaling, ...). For each transformation, display the original image, the image obtained using forward warping, and the image obtained using backward warping.