

Augmented Reality II

- Geometrical Sensor Fusion -

Gudrun Klinker

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Outline

- Stereo vision: epipolar geometry
- Combination of outside-in and inside-out tracking

Epipolar Geometry

Literature

- Richard Hartley and Andrew Zisserman, “Multiple View Geometry in Computer Vision”, Cambridge University Press, 2000. (Sections 8, 9)
- Illustrations (VRML)
at [illustrations/Epipolar/](#)

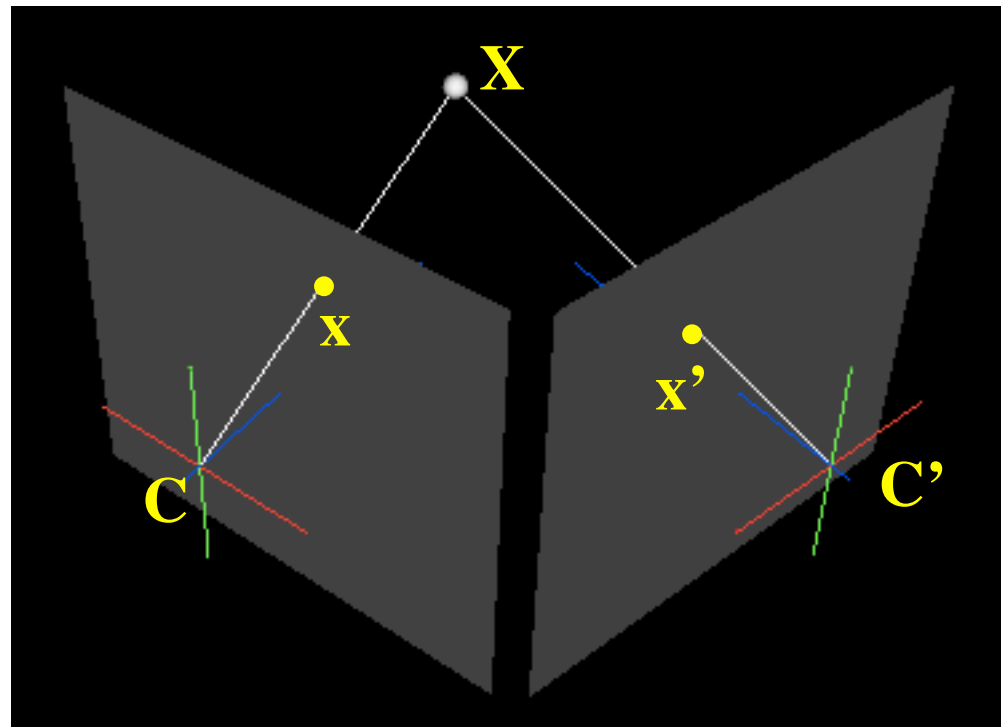
Outline

- Concepts of epipolar geometry
- Fundamental matrix F

Epipolar Geometry

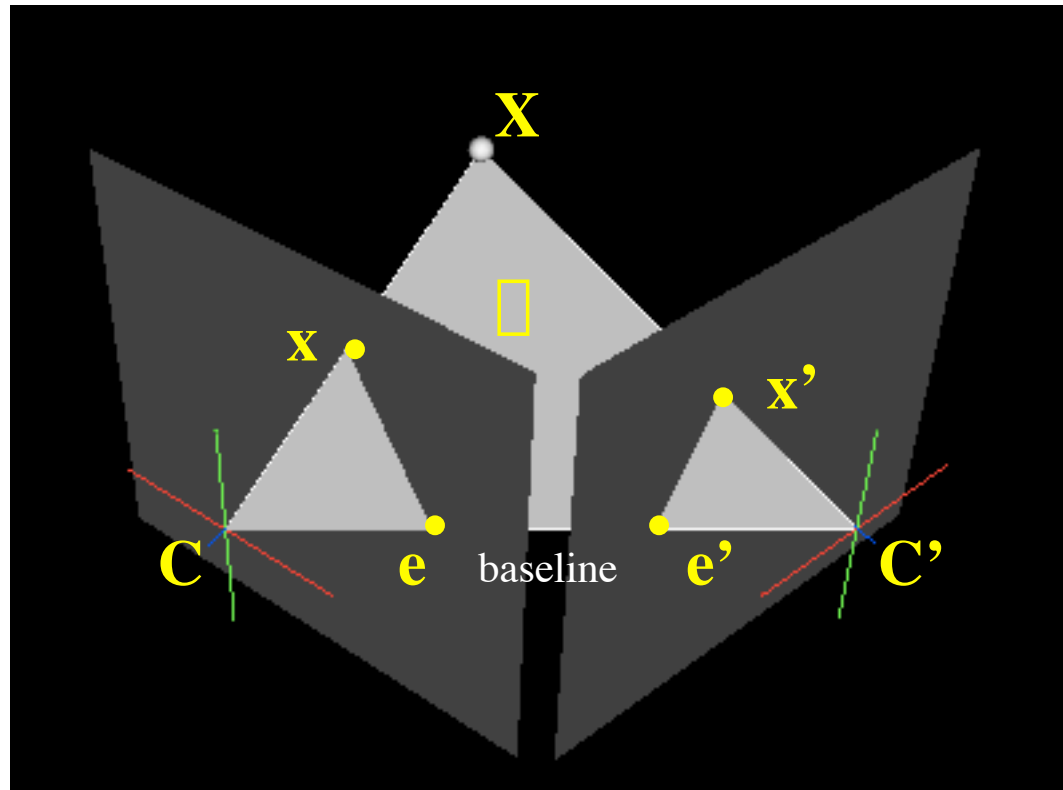
- Two Cameras:
 C and C'
- 3D-point X
- Projected 2D
image points x , x'

What are the
relationships
between these
parameters?



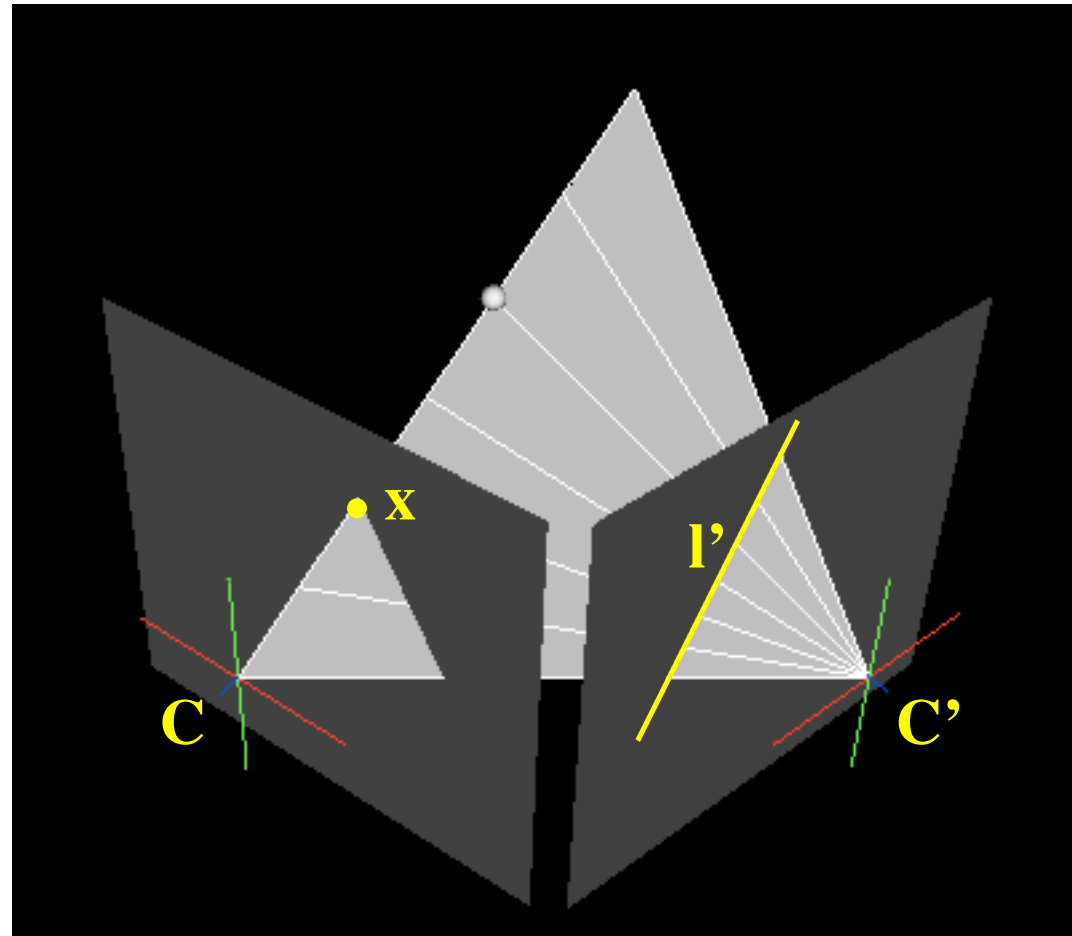
Epipolar Geometry

- Epipolar plane Π :
 - defined by C, C', X
- Baseline:
 - line between camera centers C, C'
 - all epipolar planes intersect at the baseline
- Epipoles e, e' :
 - intersection of baseline with images
 - image in one view of the camera center of the other view
 - vanishing point of the baseline direction



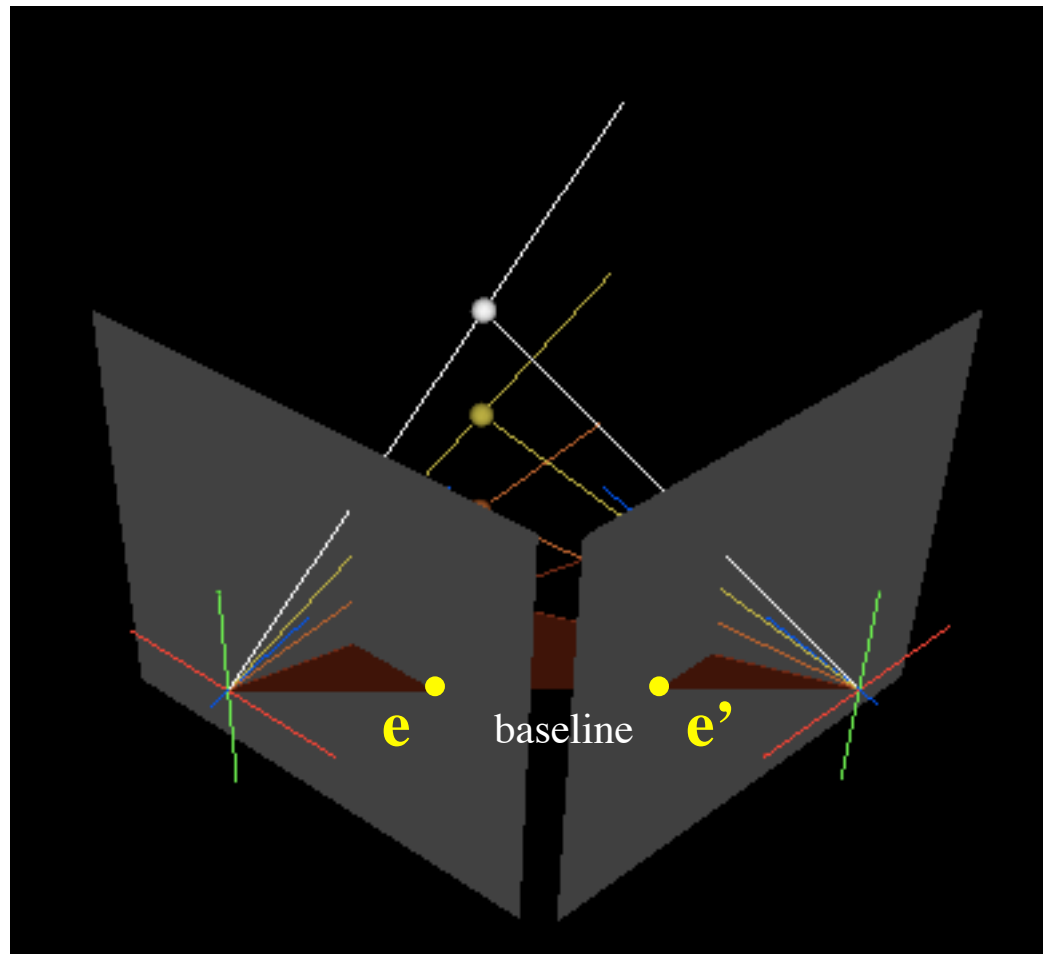
Epipolar Geometry

- Epipolar line l' for x :
 - intersection of the epipolar plane with an image plane
 - all points x' in img' which can correspond to point x in the other image



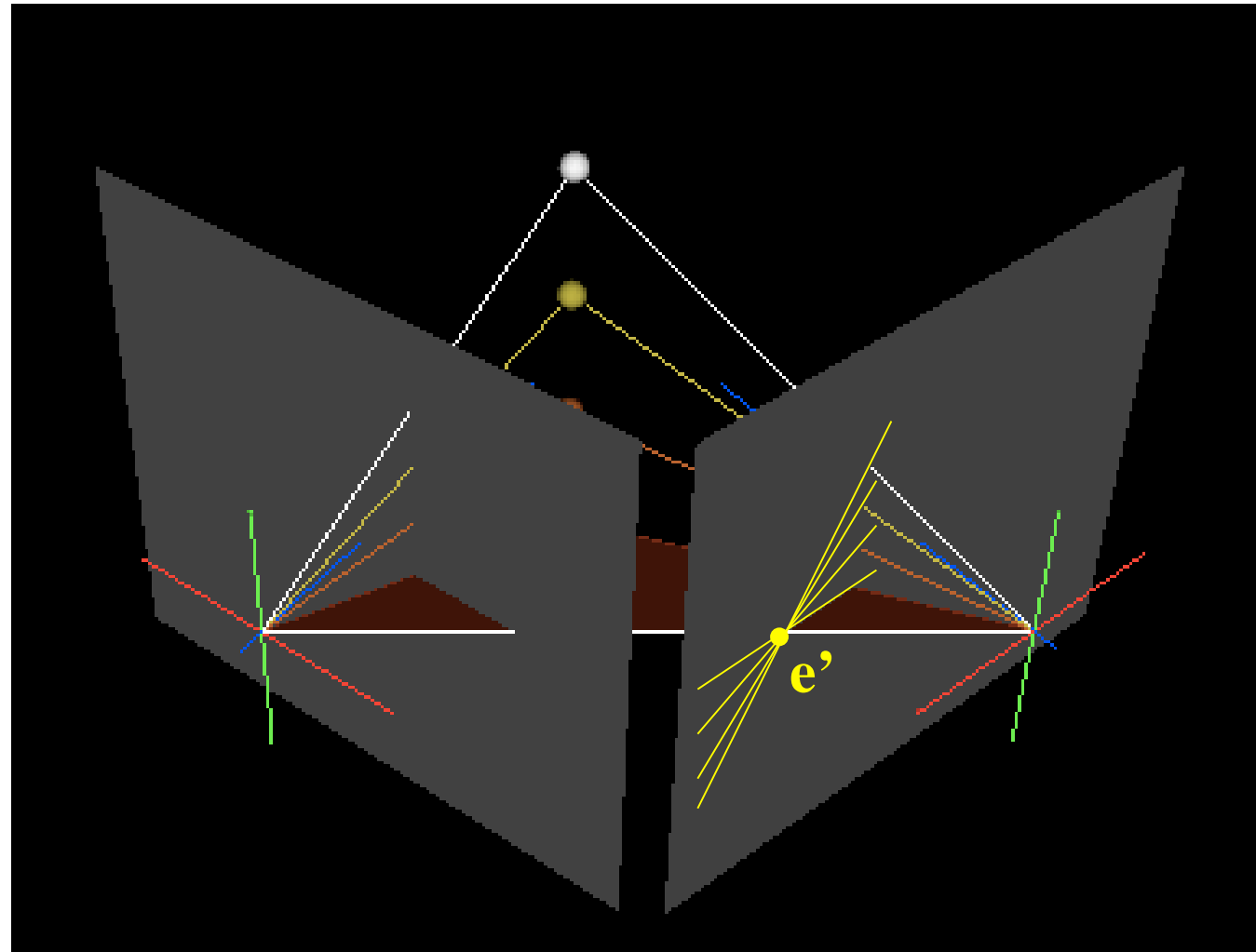
Epipolar Geometry

- Pencil of epipolar planes



Epipolar Geometry

- Pencil of epipolar lines l'



Fundamental Matrix F

- mapping: $\mathbf{x}_i \rightarrow \mathbf{l}_i'$
- \mathbf{l}_i' is a line between \mathbf{e}' and \mathbf{x}_i'
- 3D plane \square with points \mathbf{X}_i
- 2D homography H_\square mapping $\mathbf{x}_i \rightarrow \mathbf{x}_i'$
- “transfer mapping”

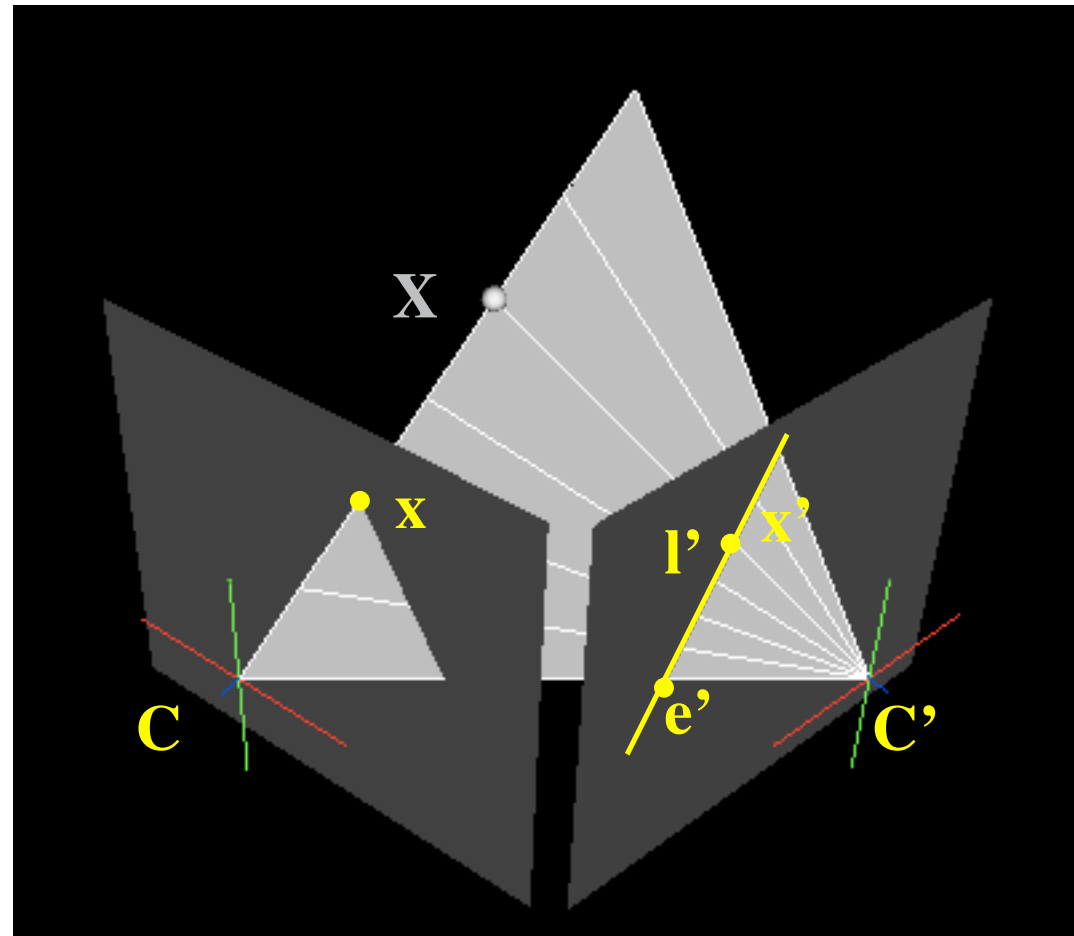
- $\mathbf{l}' = \mathbf{e}' \square \mathbf{x}'$

$$= [\mathbf{e}']_\square \mathbf{x}' \quad (\text{skew-symmetric matrix, p. 554})$$

$$= [\mathbf{e}']_\square H_\square \mathbf{x}$$

$$\mathbf{l}' = \mathbf{F}\mathbf{x}$$

- F is 3x3 matrix of rank 2

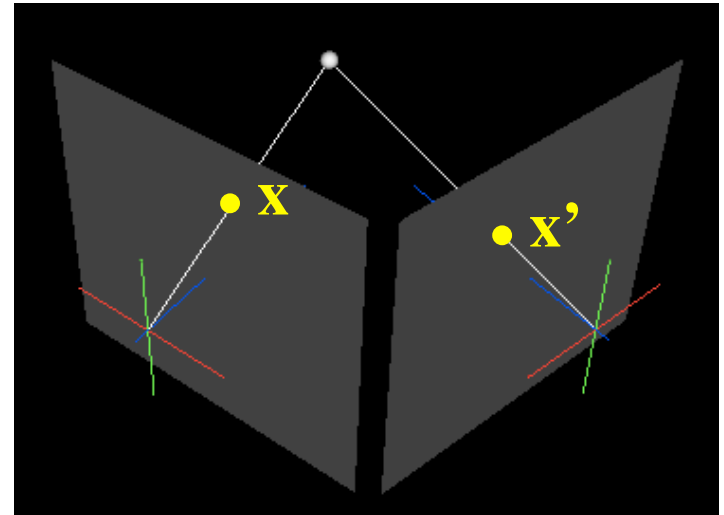


Fundamental Matrix F

- Correspondence Condition -

- For any pair of corresponding points $\mathbf{x} \iff \mathbf{x}'$:

$$\mathbf{x}'^T \mathbf{F} \mathbf{x} = 0$$



Fundamental Matrix F

- Summary -

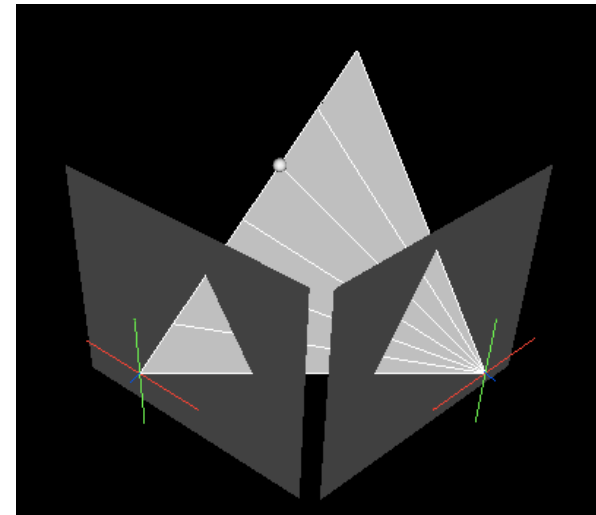
- F is of rank 2
- Point correspondence: $\mathbf{x}'F\mathbf{x} = 0$
- Epipolar lines: $\mathbf{l}' = F\mathbf{x}$

$$\mathbf{l} = F^T \mathbf{x}'$$

- Epipoles: $F\mathbf{e} = \mathbf{0}$ All epipolar lines intersect at \mathbf{e} , or \mathbf{e}' , resp.

$$F^T \mathbf{e}' = \mathbf{0}$$

- Computation from matrices P, P' : $F = [\mathbf{e}']_{\square} P' P^+$



Combination of Outside-In and Inside-Out Tracking

Literature

- William A. Hoff, “Fusion of Data from Head-Mounted and Fixed Sensors”, Proc. of the 1st International Workshop on Augmented Reality (IWAR), pp. 167-182, 1998.

Outline

- Problem statement
- Approach
- Setup
- Modeling Uncertainty

The Problem

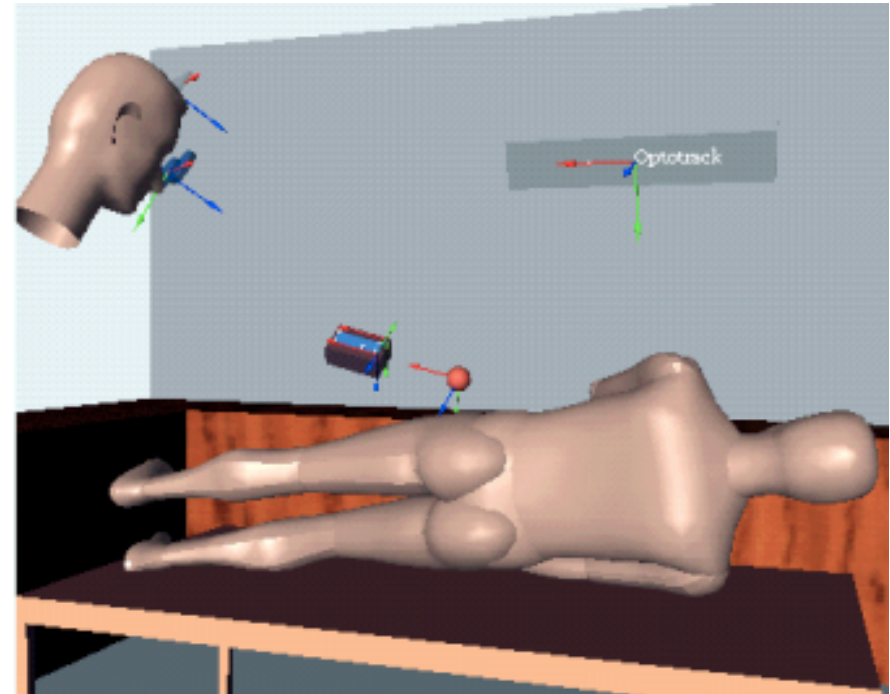
- Pose estimation depends on noisy data.
- Outside-in (stereo):
 - Precision depends on the length of the baseline
 - Position parameters: ok
 - Orientation parameters: weak
- Inside-out (mono or stereo):
 - Only short baseline available
 - Position parameters (esp. z): weak
 - Orientation parameters: ok

The Approach

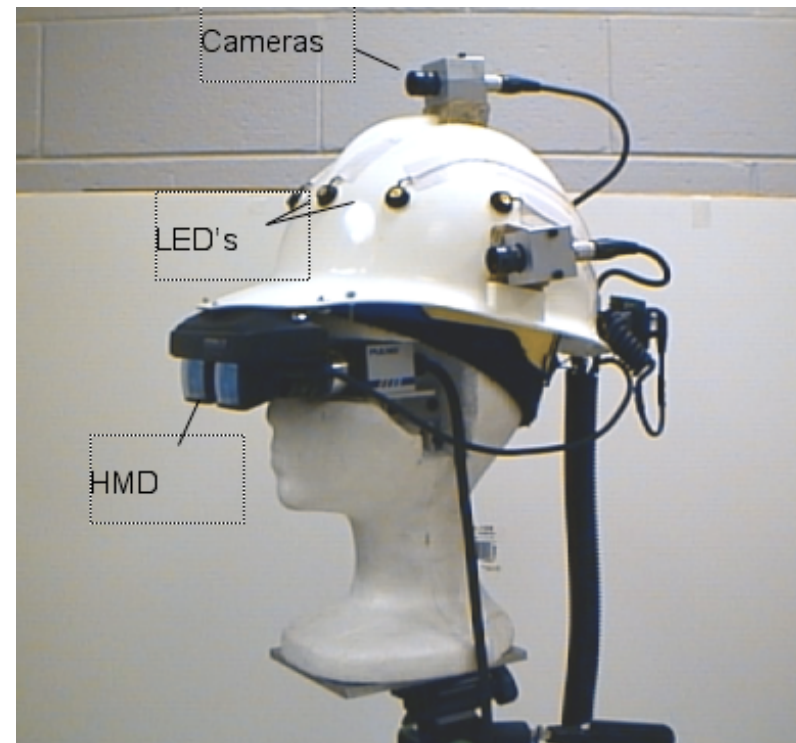
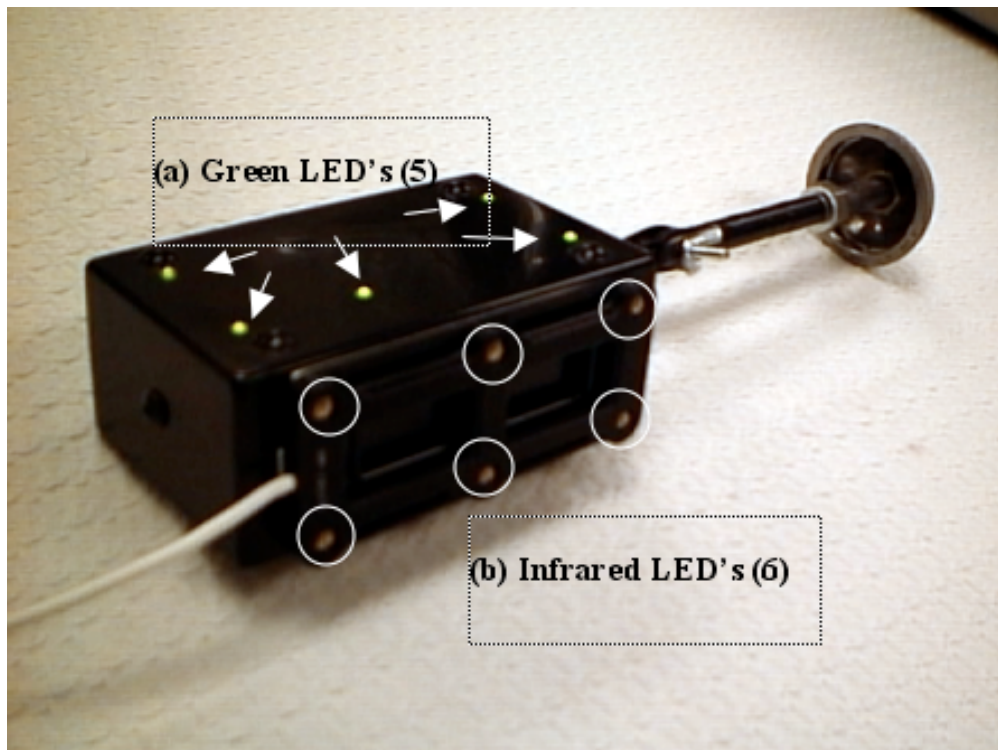
- Combine Outside-In with Inside-Out techniques to reduce noise.
- “Geometrical sensor fusion”:
exploit projection properties from different viewing angles onto the same target.
- Analog to stereo computer vision.

The Setup

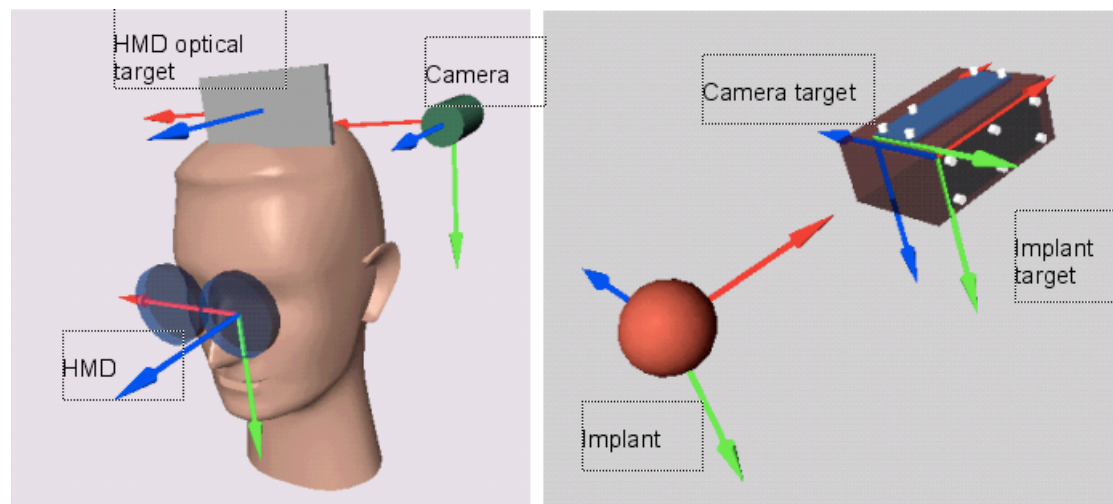
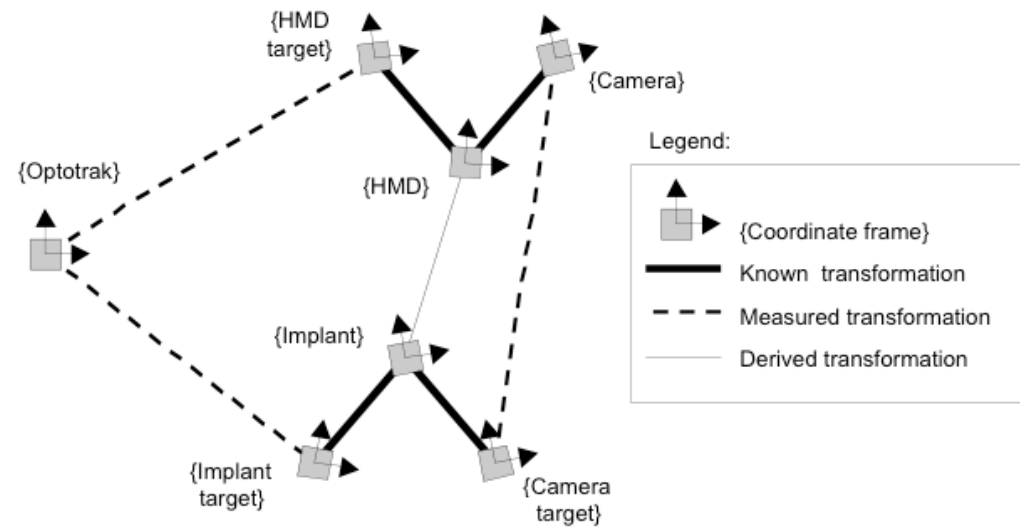
- Medicine: hip replacement



HMD and Instrument with Targets



Coordinate Systems



Modeling Uncertainty

- Pose parameters: $X=(x,y,z, r_x,r_y,r_z)$
- Noise: 6x6 covariance matrix C
- Combination of measurements:

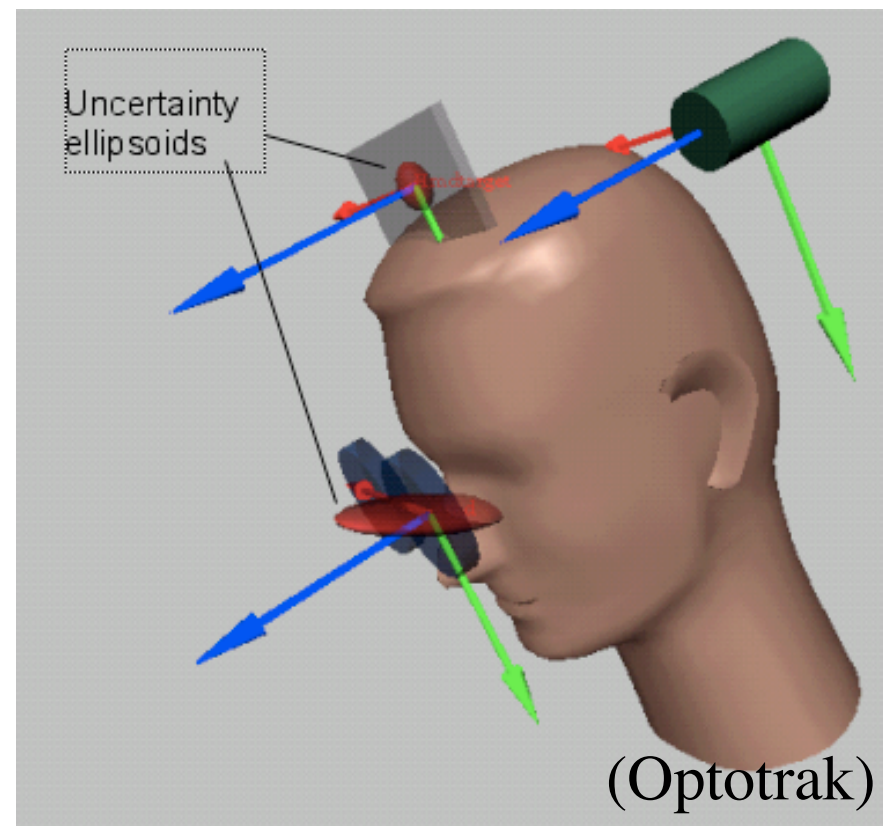
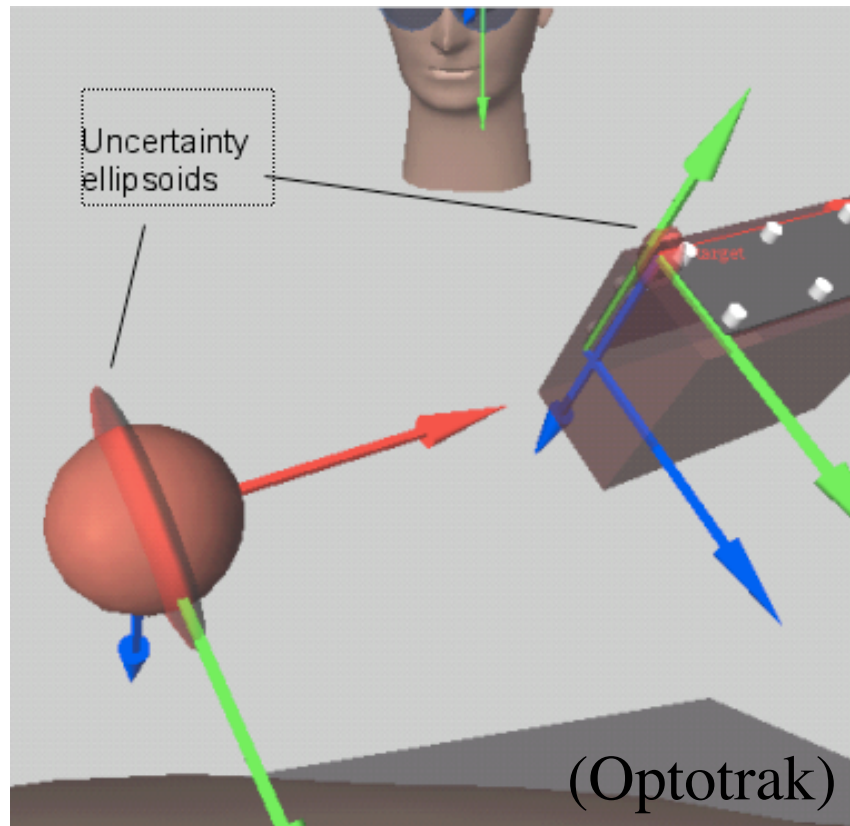
$$X = \frac{C_2}{C_1 + C_2} X_1 + \frac{C_1}{C_1 + C_2} X_2$$

$$C = \frac{C_1 C_2}{C_1 + C_2}$$

- Positional uncertainty: 3D ellipsoid

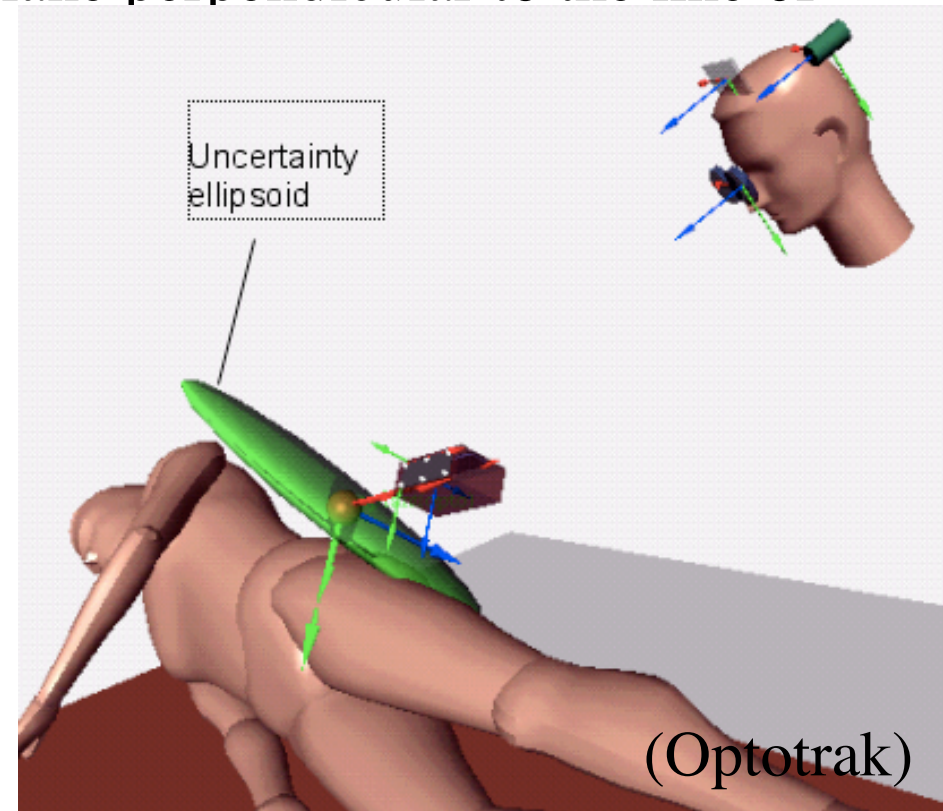
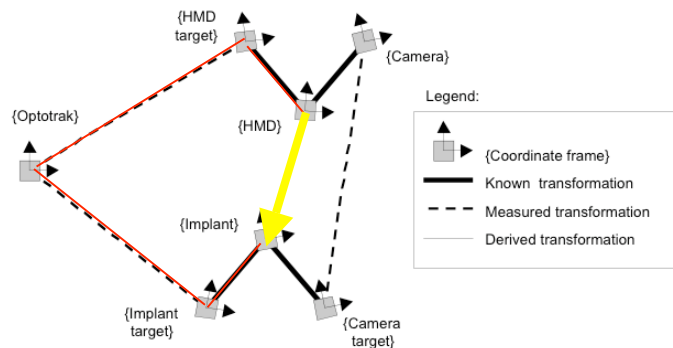
Positional Uncertainty: Optotrak

- $H_{\text{Optotrak-to-HMD}} = H_{\text{Optotrak-to-HMD-Target}} \circ H_{\text{Target-to-HMD}}$
- $H_{\text{Optotrak-to-Implant}} = H_{\text{Optotrak-to-Implant-Target}} \circ H_{\text{Target-to-Implant}}$



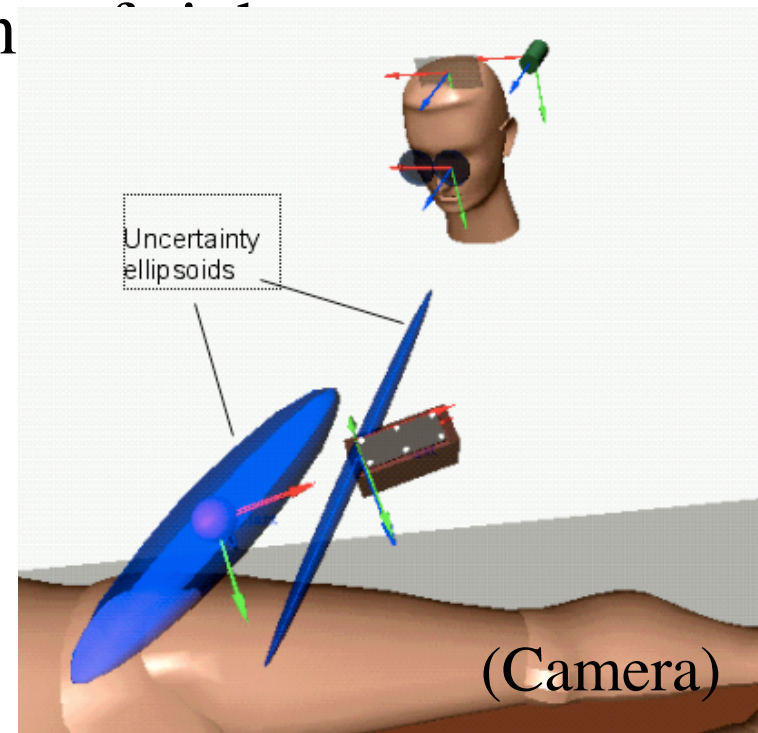
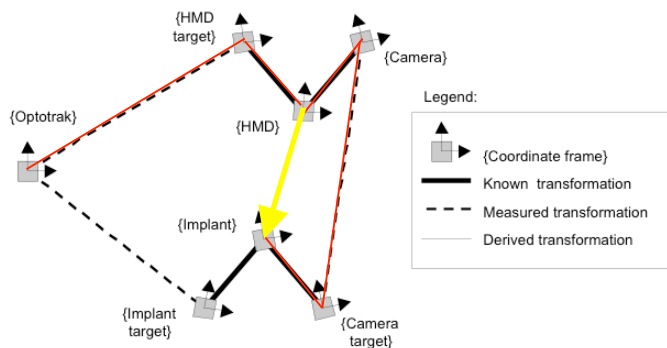
Implant Pose (Outside-In)

- Using only the Optotrak sensor
- $H_{\text{HMD-to-Implant}} = H_{\text{HMD-to-Optotrak}} \circ H_{\text{Optotrak-to-Implant}}$
- Ellipsoid elongated in the plane perpendicular to the line of sight (HMD)



Implant Pose (Inside-Out)

- Optotrak determines the HMD-mounted camera
- Camera to determines the implant target
- $H_{\text{Camer-to-Implant}} = H_{\text{Camera-to-Camera-Target}} \circ H_{\text{Camera-Target-to-Implant}}$
- Ellipsoid elongated along the line



Implant Pose (Combined)

