

Image Deconvolution

Parameter Estimation in Linear Inverse Problems

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Contents

Why not ML?

A naive approach...

...with disastrous result

Robust Parameter Estimation

Overview

Hardware Adaptation

Results

Inverse Problems

A class of its own?

- ▶ Estimating 'cause' giving rise to observed 'effect'
- ▶ Physical concepts not always meaningful
- ▶ Not a rigorous definition

Linear Inverse Problems

Special case deconvolution

Let f and g denote pristine and blurred image respectively

$$g(s) = \int_D K(s-t) f(t) dt + \nu(s)$$

where K an appropriate blur kernel and ν random noise

Discrete Model

$$g = Af + \nu$$

Parameter estimation problem: given g and A , find f !

Maximum Likelihood Estimator

Gaussian Noise

Suppose noise $\nu \sim \mathcal{N}(0, \Phi)$ is zero-mean Gaussian. Seek f s. t.

$$\log p(g|f) = c - \frac{(g - Af)^T \Phi^{-1} (g - Af)}{2} = \max!$$

Prewhitening Filter

A simple transformation...

$$g' = \Phi^{-1/2} g \qquad A' = \Phi^{-1/2} A$$

Maximum Likelihood Estimator

ML \equiv LS

...yields an equivalent Least Squares Problem

$$\arg \max \log p(g|f) = \arg \min \|g' - A'f\|_2$$

Solution

$$\hat{f}_{ML} = A'^+ g'$$

is the ML and BLU (Best Linear Unbiased) Estimator!

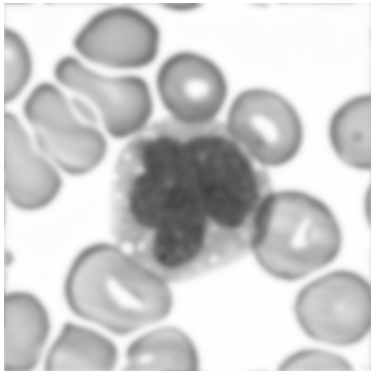
If it is that simple...

Then why have someone study this class of problems during 6 months?

- ▶ waste of manpower ($\approx 1000\text{h!}$)
- ▶ waste of resources (yet another blocked MATLAB-License...)
- ▶ waste of money (at least I was cheap...)

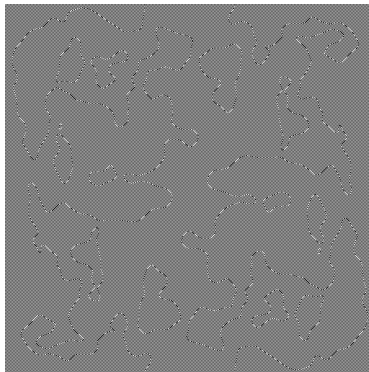
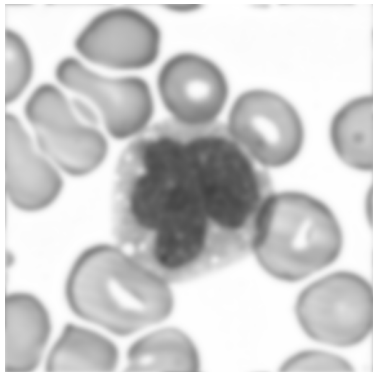
What it's really worth...

Blurred with Gaussian kernel



What it's really worth...

Blurred with Gaussian kernel



ML Estimate and BLUE

(?)

Not quite so simple, then...

Ok, what went wrong here?

- ▶ Inverse computed by FFT
- ▶ Orthogonal Transform
- ▶ Numerically as well-behaved as can be ($\kappa = 1$)
- ▶ Rounding error $\epsilon \approx 2 \times 10^{-16}$

Problem: Amplification of Noise

For most cases of practical interest $\text{Var}(\hat{f}_{ML}) \approx \infty$

Inverse Problems need Regularization!

Robust Parameter Estimation

What we *cannot* do

- ▶ Noisy environment provokes loss of information
- ▶ Complete restoration not a realistic expectation

In what sense 'robust', then?

- ▶ Ensure that deterioration is gradual rather than abrupt
- ▶ Make solution depend continuously on the data

Tarantola, Inverse Problem Theory and Methods for Model Parameter Estimation, 2005

'Classical' Techniques

Mean (Integrated) Square Error

Construct estimator for **deterministic parameter** f with minimal

$$MISE(\hat{f}) = E \left\{ \|\hat{f}(g) - f\|_2^2 \right\} = \|\text{Bias}(\hat{f})\|_2^2 + \text{trace}(\text{Var}(\hat{f}))$$

Idea: trade variance for bias

Accept systematic error, obtain stability in turn

$$\underbrace{\begin{pmatrix} 0 \\ \infty \end{pmatrix}}_{\hat{f}_{ML} = A^+} \quad \longleftarrow \quad \underbrace{\begin{pmatrix} \text{Bias}(\hat{f}) \\ \text{Var}(\hat{f}) \end{pmatrix}}_{\hat{f}_{mmse} = ?} \quad \longrightarrow \quad \underbrace{\begin{pmatrix} f \\ 0 \end{pmatrix}}_{\hat{f}_{triv} \equiv 0}$$

Open questions and where to look for answers...

How to parameterize Trade-Off-Curve?

→ Filter Functions

- ▶ Ridge Regression (Tikhonov)
- ▶ Truncated SVD ('Spectral-Cut-Off')

Where to look for a minimizer?

→ Parameter Rules

- ▶ Generalized Cross Validation (GCV)
- ▶ L-Curve

Wahba, Practical Approximate Solutions to Linear Operator Equations, 1977

Hansen, The L-Curve and its Use in the Numerical Treatment of Inverse Problems, 2001

Bayesian Approach

Recast the Problem

Model the parameter f as a random variable with (non-flat) pdf

Optimality of the Conditional Mean

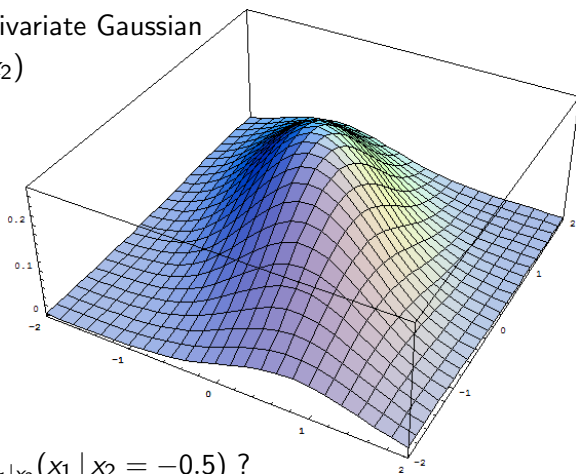
Let $\hat{f}_{mmse}(g) := E\{f|g\}$. Then for any estimator \hat{f}

$$MISE(\hat{f}) \geq MISE(\hat{f}_{mmse})$$

Special Case: Multivariate Gaussian

Pdf of a bivariate Gaussian

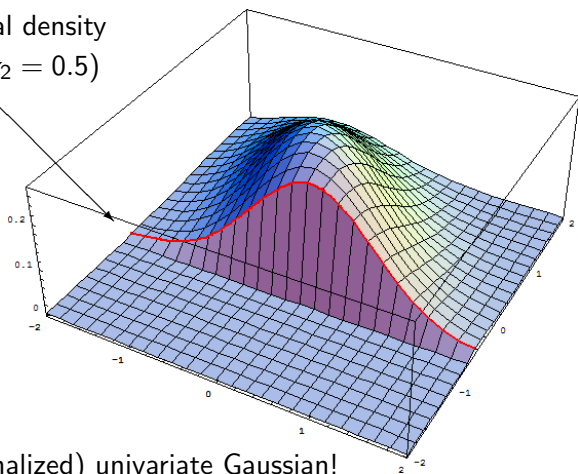
$$X = (x_1, x_2)$$



What is $f_{x_1|x_2}(x_1 | x_2 = -0.5)$?

Special Case: Multivariate Gaussian

Conditional density
 $f_{x_1|x_2}(x_1 | x_2 = 0.5)$



is (unnormalized) univariate Gaussian!

Special Case: Multivariate Gaussian

Conditional Mean of a multivariate Gaussian

Let f, g be two random variables with jointly Gaussian pdf

$$\begin{pmatrix} f \\ g \end{pmatrix} \sim \mathcal{N} \left(\begin{pmatrix} \mu_f \\ \mu_g \end{pmatrix}, \begin{pmatrix} \text{Var}[f] & \text{Cov}(f, g) \\ \text{Cov}(g, f) & \text{Var}[g] \end{pmatrix} \right)$$

Then

$$f|g \sim \mathcal{N}(\mu_{f|g}, \Phi_{f|g})$$

is also Gaussian with mean

$$\begin{aligned} \mu_{f|g} &= \underbrace{\mu_f + \text{Cov}(f, g) \text{Var}[g]^{-1} (g - \mu_g)}_{= \hat{f}_{mmse}(g)} \end{aligned}$$

Wiener Filter: Gaussian Case

Optimal Linear Estimator

Seek affine linear mapping $g \mapsto \hat{f}_W(g)$ such that

$$MISE(\hat{f}_W) = \min!$$

For the special case of Gaussian RVs

we already know that \hat{f}_{mmse} is affine linear in g , hence

$$\hat{f}_W = \hat{f}_{mmse} : g \mapsto \mu_f + \text{Cov}(f, g) \text{Var}[g]^{-1} (g - \mu_g)$$

Wiener Filter: General Case

Even in the general case

where \hat{f}_{mmse} is not affine linear, one can show that

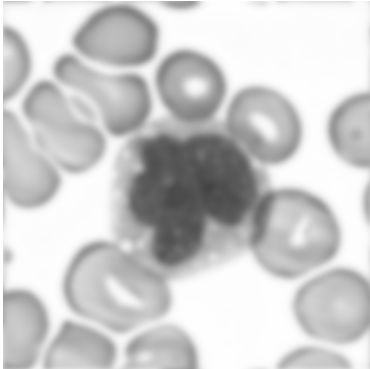
$$\hat{f}_W : g \mapsto \mu_f + \text{Cov}(f, g) \text{Var}[g]^{-1} (g - \mu_g)$$

is still the best linear approximation, even though

$$\hat{f}_W(g) \neq E\{f|g\} = \hat{f}_{mmse}(g)$$

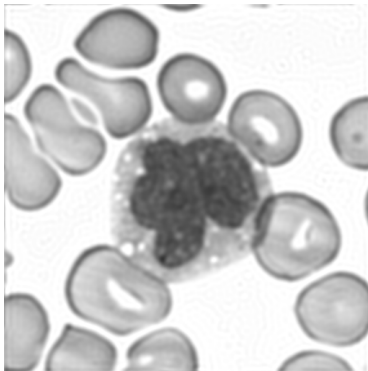
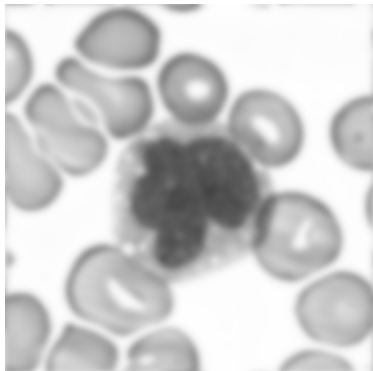
Let's test it...

Blurred with Gaussian kernel



Pretty good, isn't it?

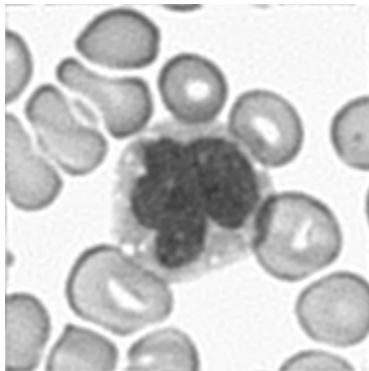
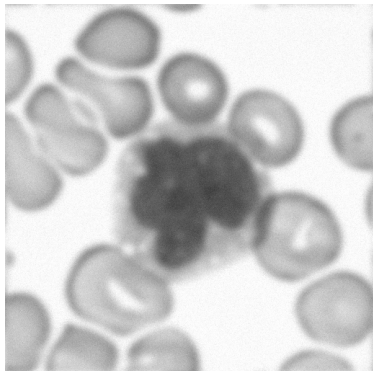
Blurred with Gaussian kernel



Wiener Estimate

Even with a decent amount of noise...

Blurred plus white noise
 $\sigma^2 = 25$ (SNR = 32.4737 dB)

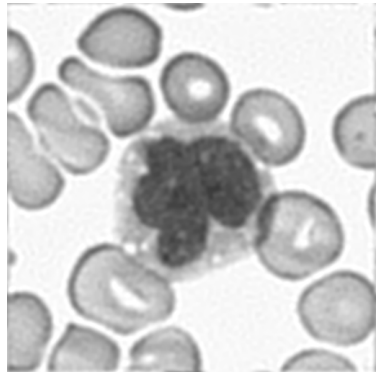
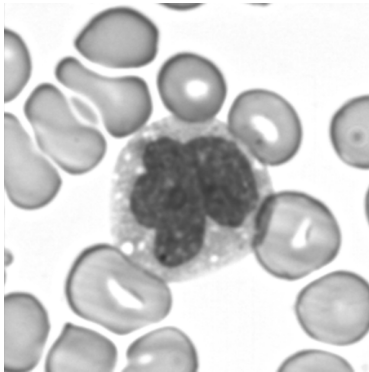


Wiener Estimate

(!)

Compare with the original

Pristine image



Wiener Estimate

(!)

Implemented Algorithms (C++)

Wiener Filter

- ▶ Direct approach

Blind Deconvolution

- ▶ EM-based, iterative
- ▶ Gaussian noise model
- ▶ Gaussian image model
- ▶ Unknown blur kernel

Richardson-Lucy

- ▶ EM-based, iterative
- ▶ Poissonian noise model

Neelamani

- ▶ Hybrid approach
- ▶ Alternate filtering in Fourier/Wavelet-domain

Katsaggelos, Digital Image Restoration, 1991

Neelamani *et al.*, ForWaRD: Fourier-Wavelet Regularized Deconvolution for Ill-Conditioned Systems, 2004

Error Metric

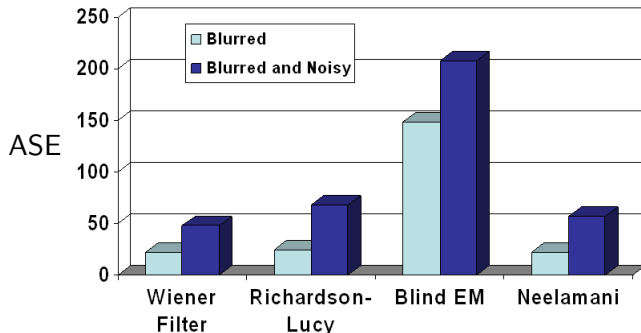
Evaluation Criterion

Assessment by square error, averaged over the spatial coordinates

$$ASE(\hat{f}) = \left\| \hat{f} - f \right\|_2^2 / \dim(f)$$

where \hat{f} is the estimate of the pristine image f

Evaluation: Results



Test-image	WF	RL	BEM	NE
Blurred	22.2589	24.3831	148.6752	21.9385
Blurred& Noisy	47.903	68.1796	207.6493	56.551

Hardware Adaptation

Model-Problem

- ▶ Solution not well-determined by the data
- ▶ Incorporation of a-priori knowledge crucial for success

Hardware

Component	Manufacturer/Specification
Microscope	Zeiss Axio Imager M1, dry
Lens	Zeiss Achromplan 63x, 0,95 numerical aperture
CCD-camera	JAI Pulnix TMC-1402 CL (1392x1040 and 800x600)

CCD-Camera Noise: Covariances

Given $k = 100$ iid darkframe images, estimate covariances!

Sample covariance matrix (=sum of rank-one updates)

$$\bar{C} = \frac{1}{k-1} \sum_{i=1}^k (\nu_i - \bar{\nu})(\nu_i - \bar{\nu})^T$$

Problem

For high-dimensional ν cov. are poorly determined by 100 samples!

$$\text{rank}(\bar{C}) \leq k \ll \dim(\nu) = 800 \times 600$$

CCD-Camera Noise: Covariances

Idea: Use additional 'information'

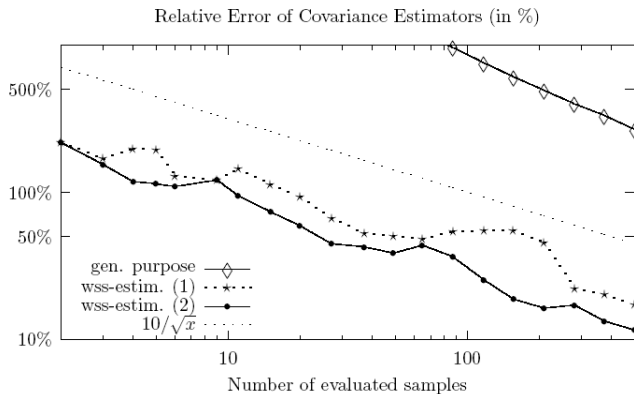
Wide-Sense-Stationarity (WSS) assumption

- ▶ Constant mean
- ▶ Correlation is function of 2D-distance
- ▶ $\text{Var}[\nu]$ is Block-Toeplitz with Toeplitz Blocks (BTTB)

Agenda

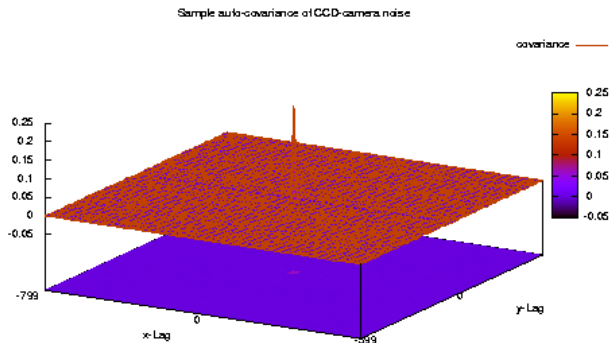
Construct unbiased estimator for WSS-random-processes!

Test on synthetically created data



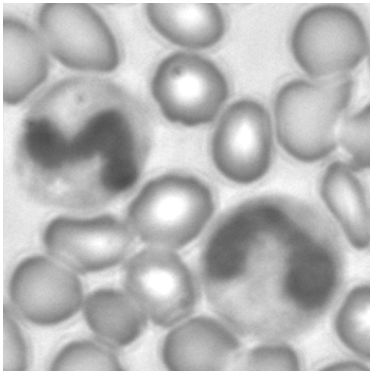
Application to darkframe images

Auto-Covariance of CCD-camera noise



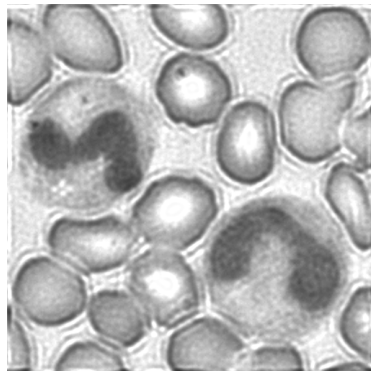
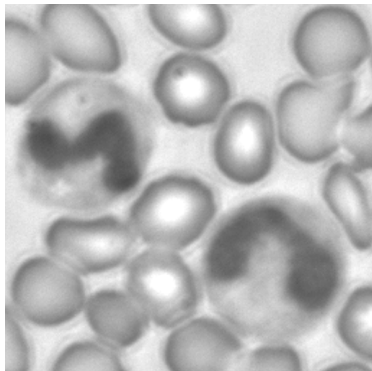
Test with 'authentic' data

Recorded image



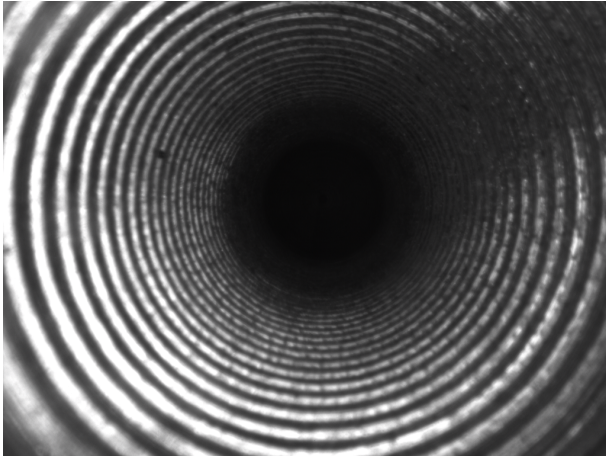
Test with 'authentic' data

Recorded image

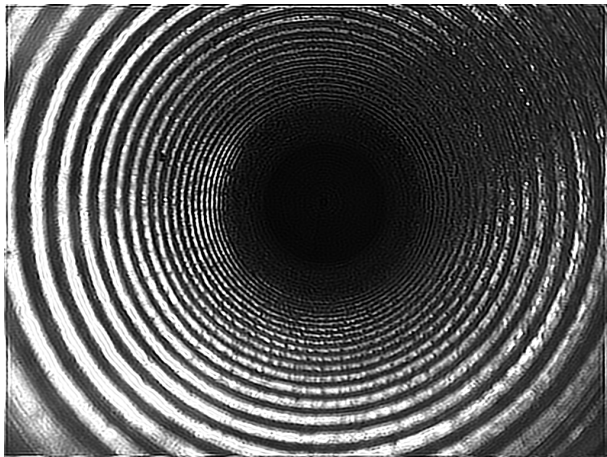


Richardson-Lucy estimate

Application to endoscopic imaging (recorded)



Application to endoscopic imaging (deconvolved)



Conclusion

Linear Inverse Problems

- ▶ Well-investigated class of problems
- ▶ Particularly simple, because linear
- ▶ Practical relevance for a wide range of applications

Suggested further reading :-)

<http://home.cs.tum.edu/~kasper/da/thesis.pdf>