

Manifold Learning for Shape Retrieval

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Manifold Learning Tutorial

September 22, 2011

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Key Concepts

1. Preliminaries: Manifold Learning Review
2. Constructing Shape-based Manifolds
3. Manifold Learning for Content-based Image Retrieval (CBIR)

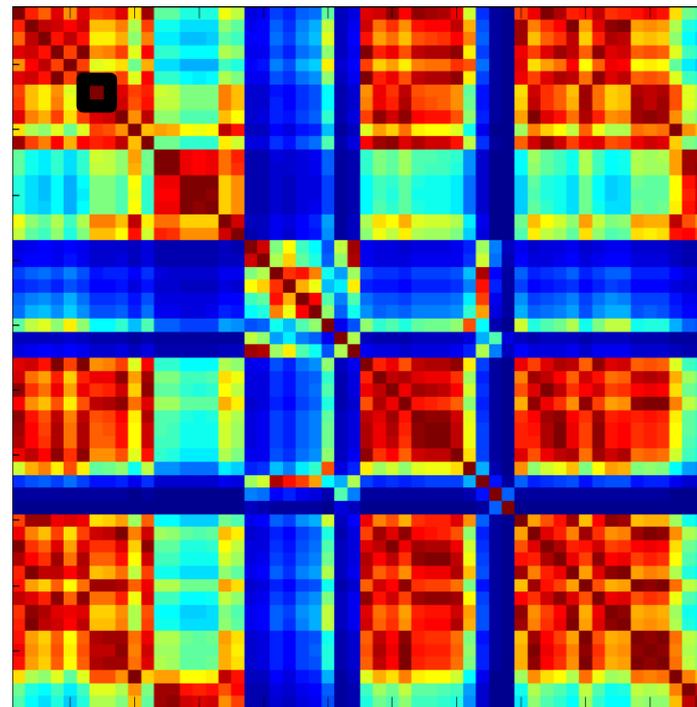
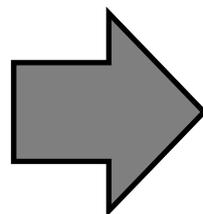
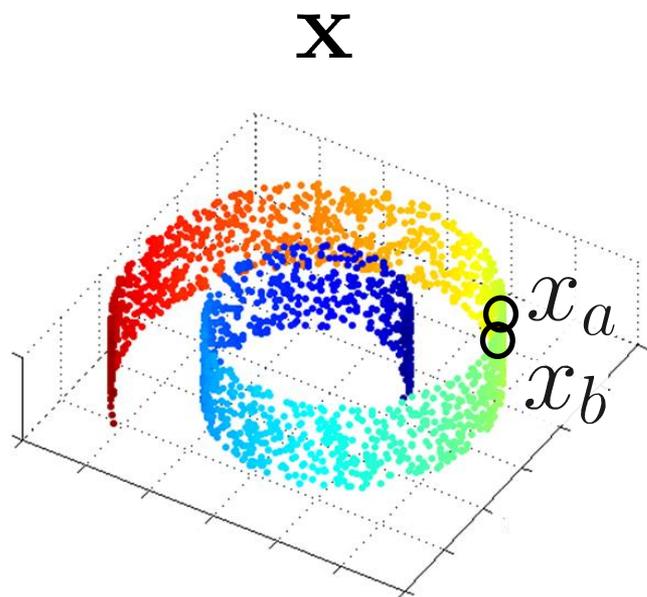
Preliminaries: Manifold Learning Review

- For high dimensional dataset x learn low dimensional embeddings y
- Low dimensional embeddings preserve high dimensional structure
 - Similar samples near in embedding space
 - Dissimilar samples are far away in embedding space

Preliminaries: Graph Embedding (GE)

Step 1: Calculate Similarity Matrix W

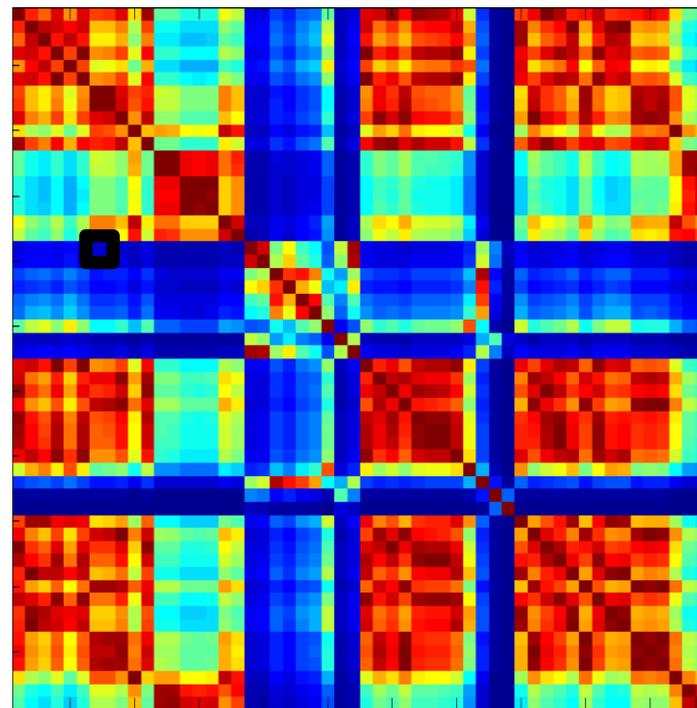
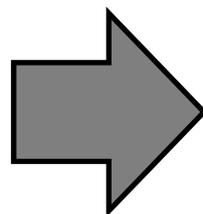
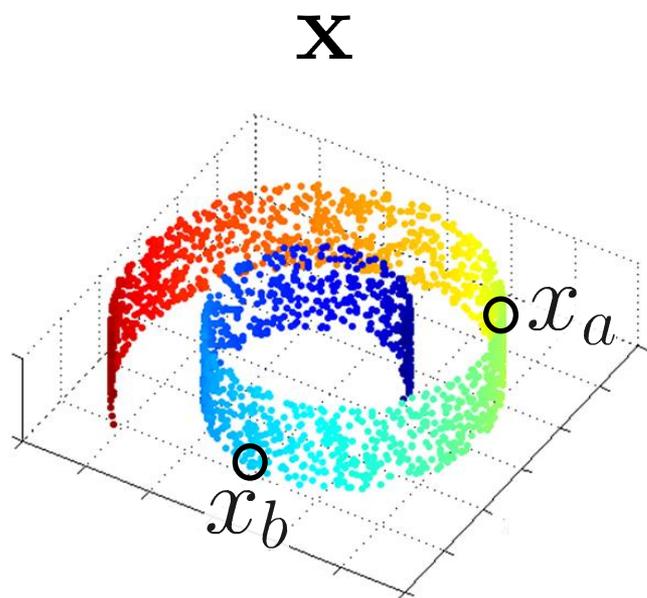
- Similar samples: high value



Preliminaries: Graph Embedding (GE)

Step 1: Calculate Similarity Matrix W

- Dissimilar samples: low values

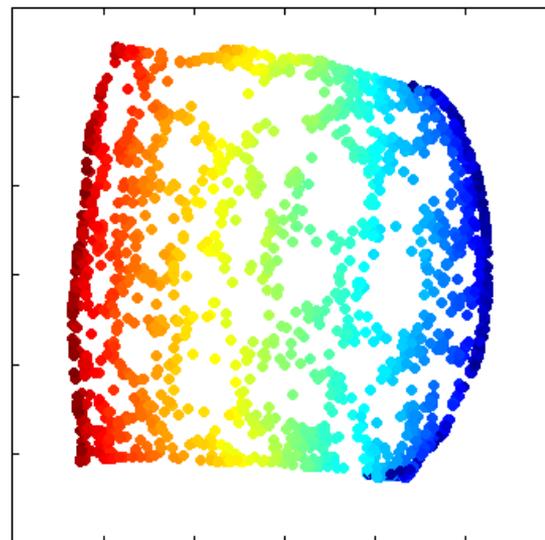
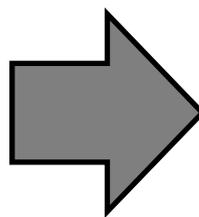
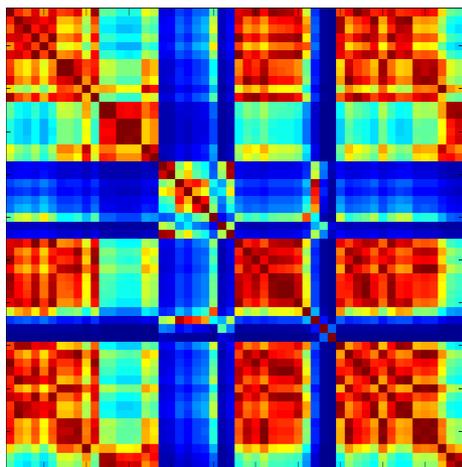


Preliminaries: Graph Embedding (GE)

Step 2: Calculate Embeddings:

- Minimize distance between similar samples:

$$\mathbf{y} = \underset{\mathbf{y}}{\operatorname{argmin}} \sum_{a=1}^N \sum_{b=1}^N \|y_a - y_b\|^2 W(x_a, x_b)$$

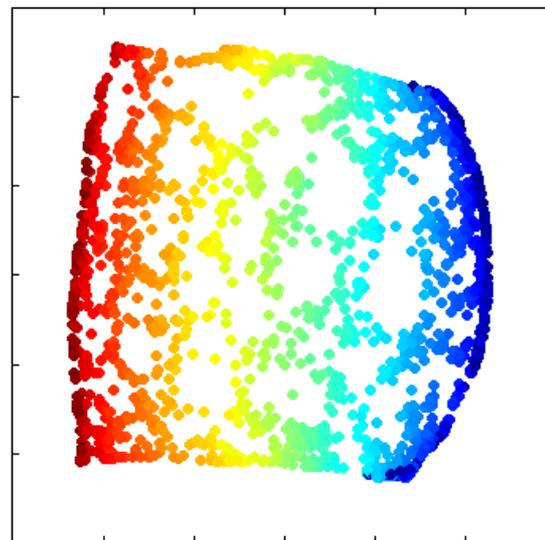
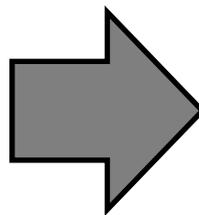
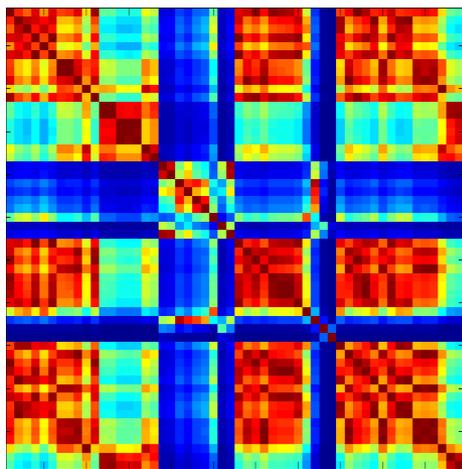


Preliminaries: Graph Embedding (GE)

Step 2: Calculate Embeddings:

- Equivalent to eigenvalue decomposition (EVD):

$$(D - W)\mathbf{y}^T = \lambda D\mathbf{y}^T$$



Key Concepts

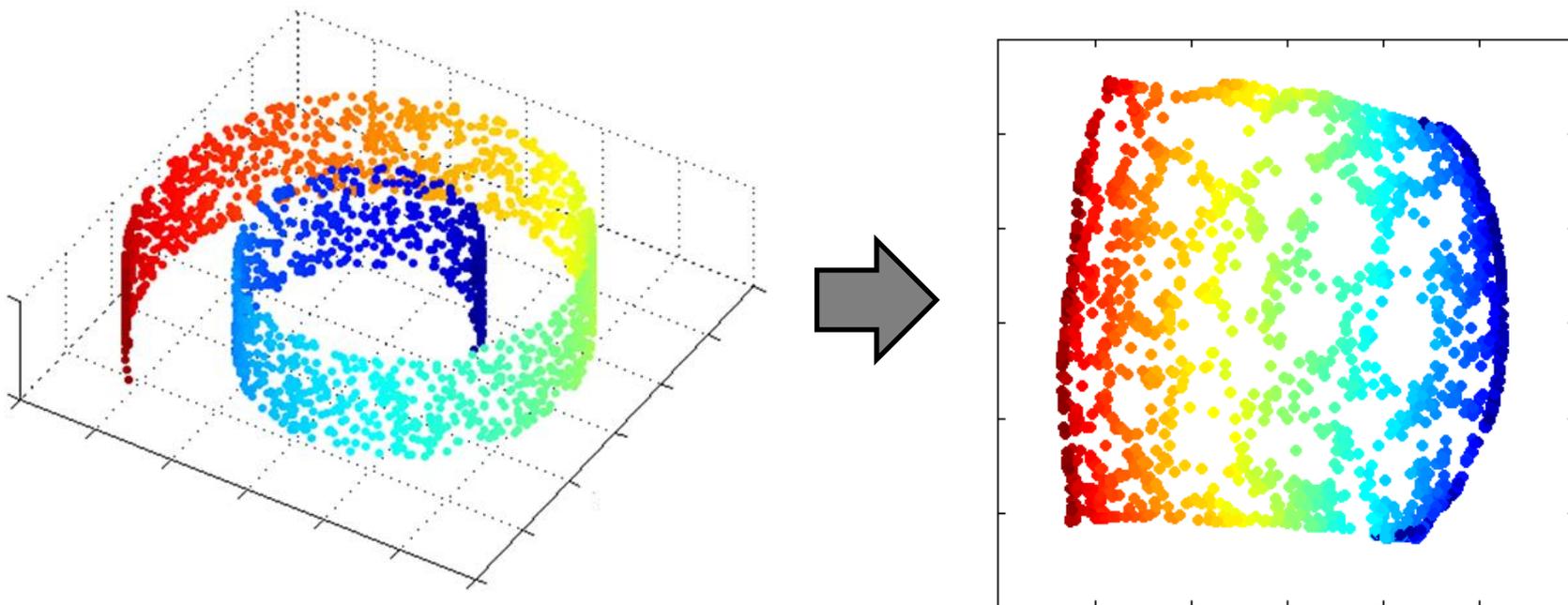
1. Preliminaries: Manifold Learning Review
2. Constructing Shape-based Manifolds
3. Manifold Learning for Content-based Image Retrieval (CBIR)

Similarity Measures

- Formulate W to describe shape similarity between medical objects of interest
- W user defined function to measure similarity
 - Must always be positive semi-definite
 - Must be symmetric

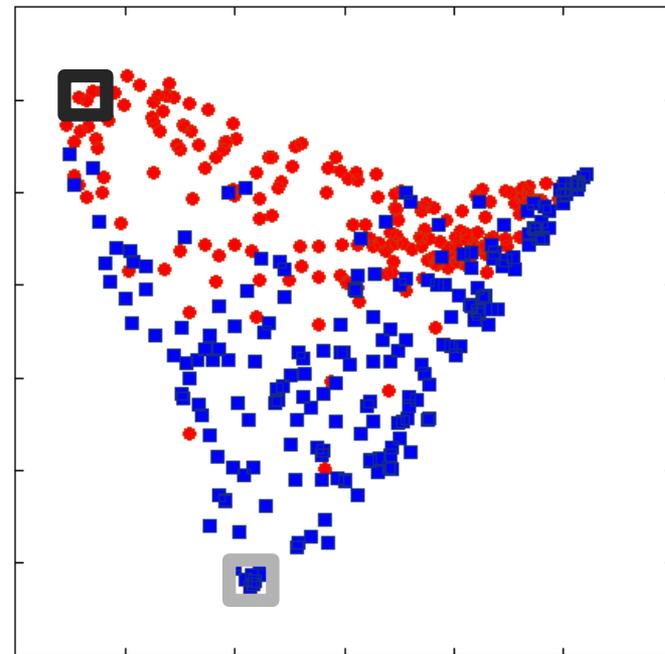
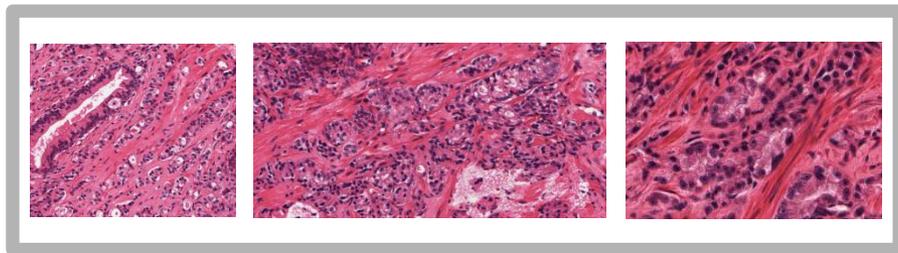
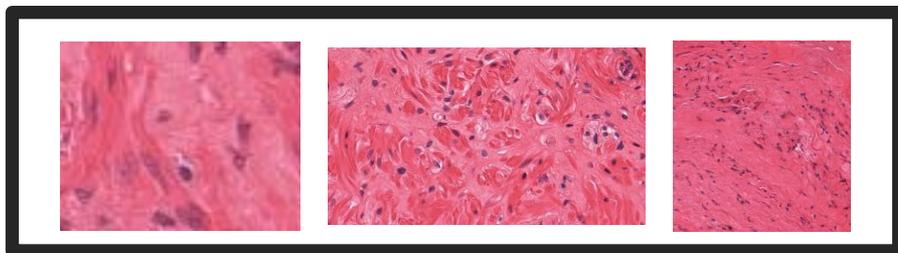
Preliminaries: Similarity Measures

- **Gaussian Kernel:** $W(a, b) = e^{-\|x_a - x_b\|/\gamma}$
 - Where x_a and x_b are pixel values from two images (Tenenbaum 2000, Roweis 2000)



Preliminaries: Similarity Measures

- **Gaussian Kernel:** $W(a, b) = e^{-\|x_a - x_b\|/\gamma}$
 - Where x_a and x_b are features (e.g. texture) extracted from two images (Doyle 2007, Basavanhally 2010)



Shape Similarity Approach 1

- **Gaussian Kernel:** $W(a, b) = e^{-\|x_a - x_b\|/\gamma}$
 - Define x_a and x_b as a set of morphological features (Naik 2008)

Shape Similarity Approach 2

- Diffeomorphic-based Similarity (DBS):

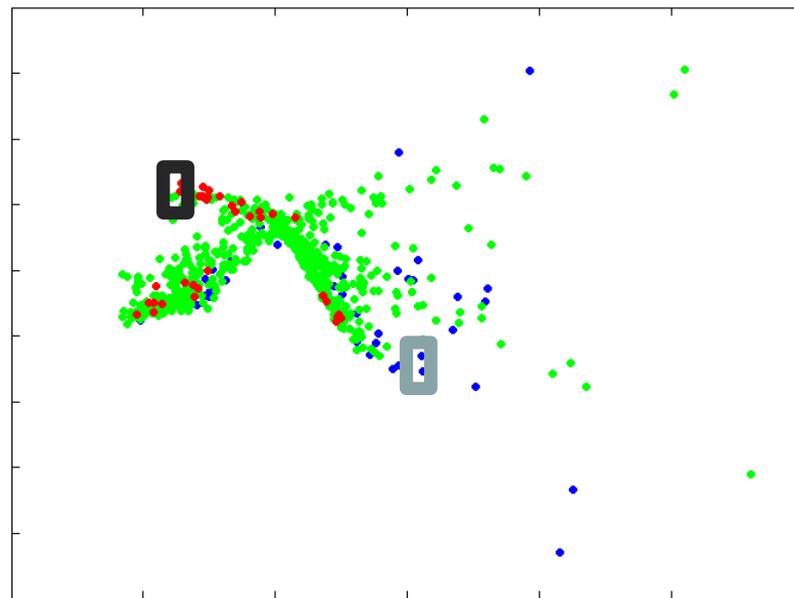
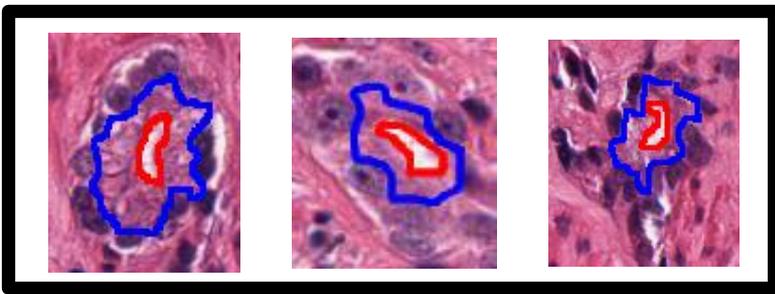
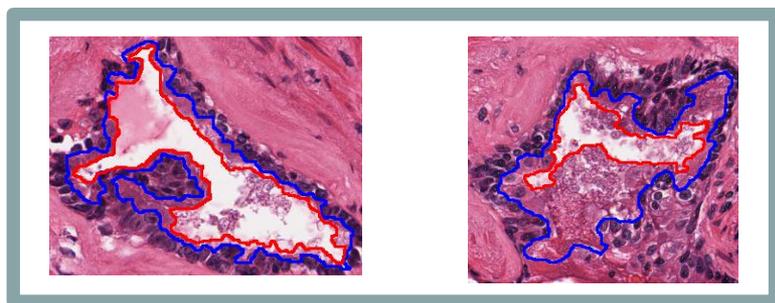
$$W(a, b) = e^{-A(o_a, o_b)/\gamma}$$

- Where o_a and o_b are objects
- A quantifies object similarity between shapes directly (Sparks 2010)

Shape Similarity Approach 2

- Diffeomorphic-based Similarity (DBS):

$$W(a, b) = e^{-A(o_a, o_b)/\gamma}$$



Shape Similarity Measures

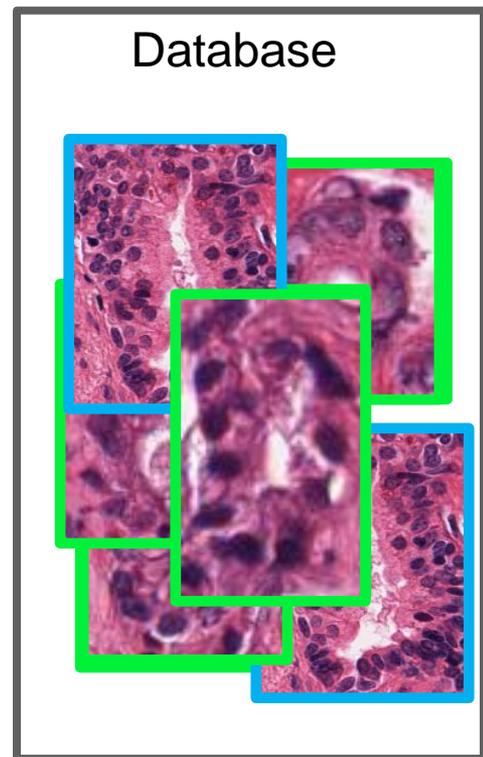
- W can describe shape similarity:
 - Comparison between morphologic features
 - Comparison between objects directly

Key Concepts

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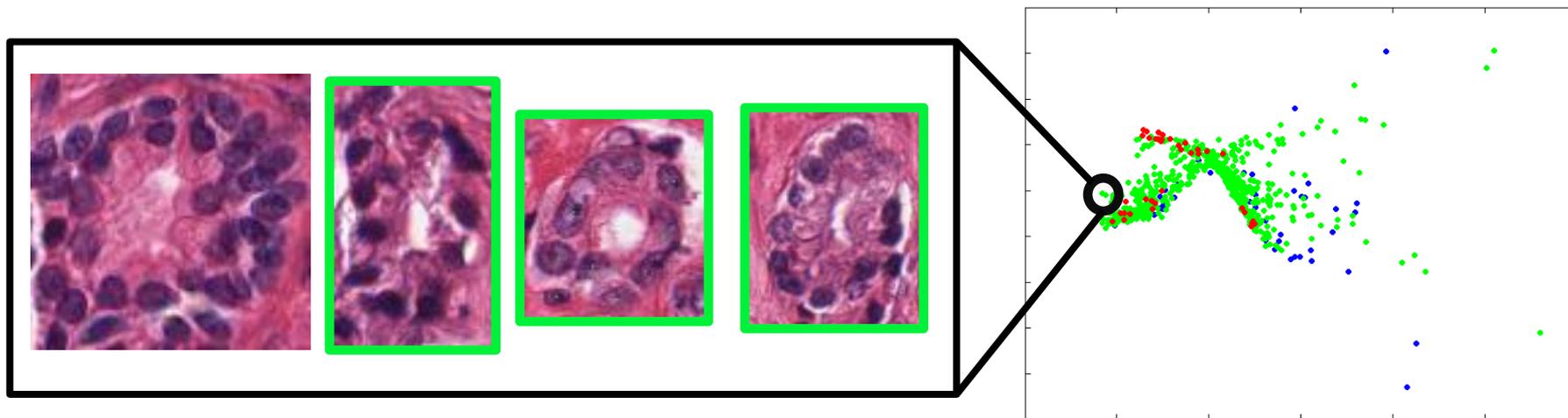
Content-Based Image Retrieval

- Want to retrieve images most similar to a selected query image

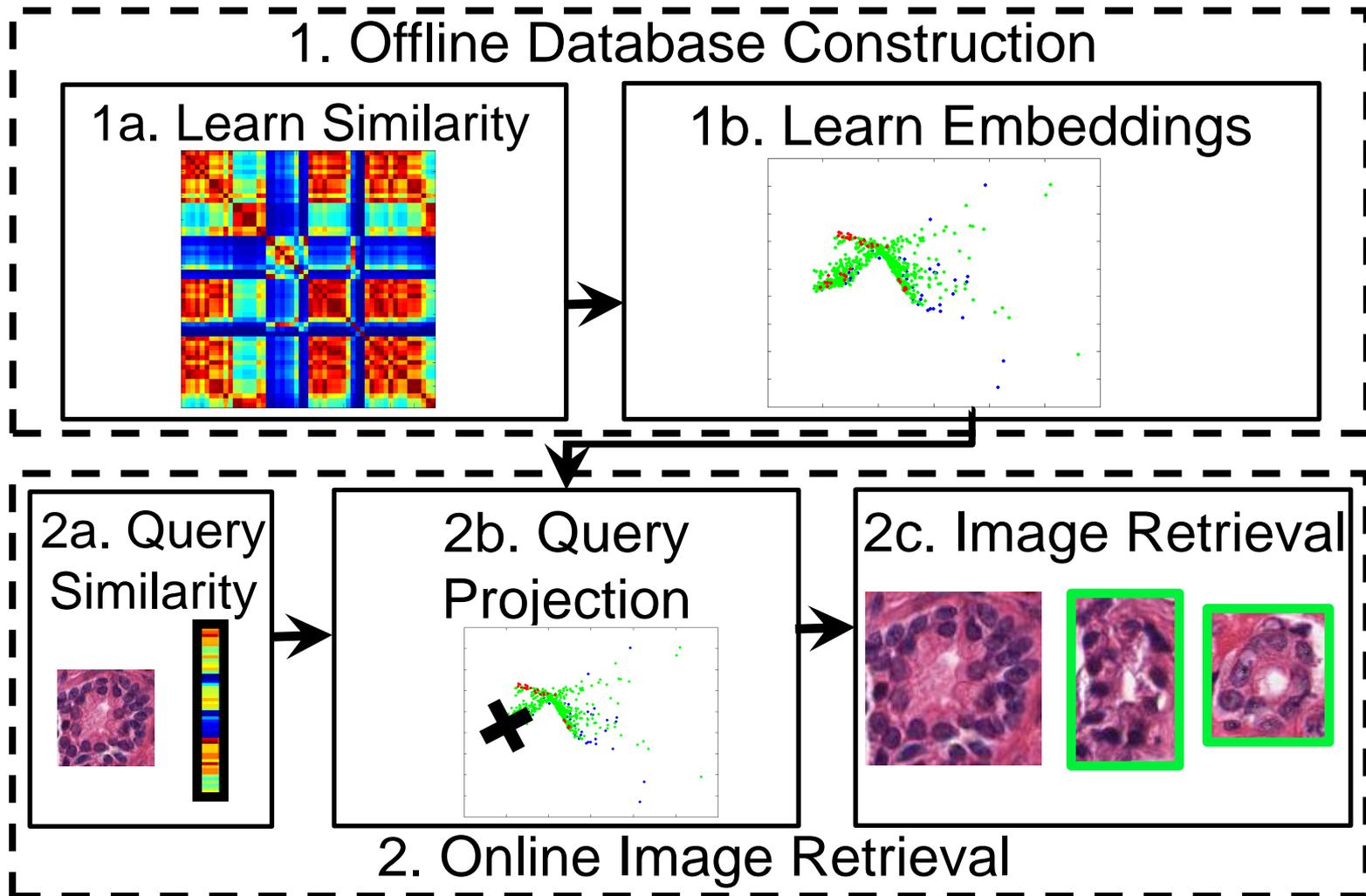


Content-Based Image Retrieval

- Want to retrieve images most similar to a selected query image
- Similar images are near each other in embedding space



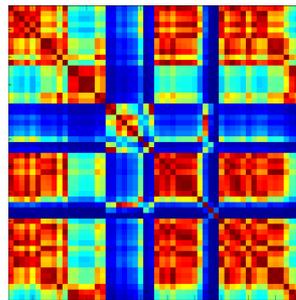
CBIR System



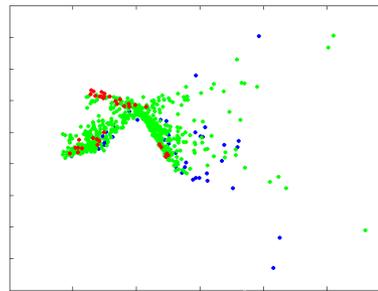
CBIR System

1. Offline Database Construction

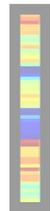
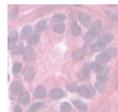
1a. Learn Similarity



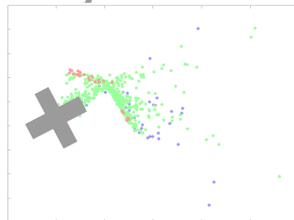
1b. Learn Embeddings



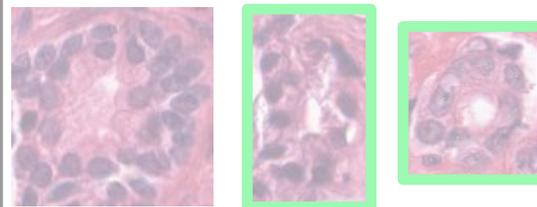
2a. Query Similarity



2b. Query Projection



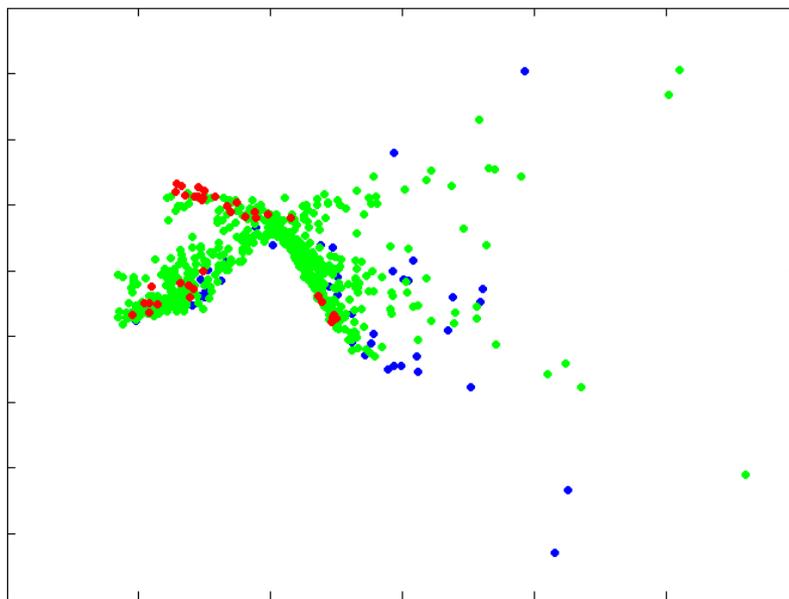
2c. Image Retrieval



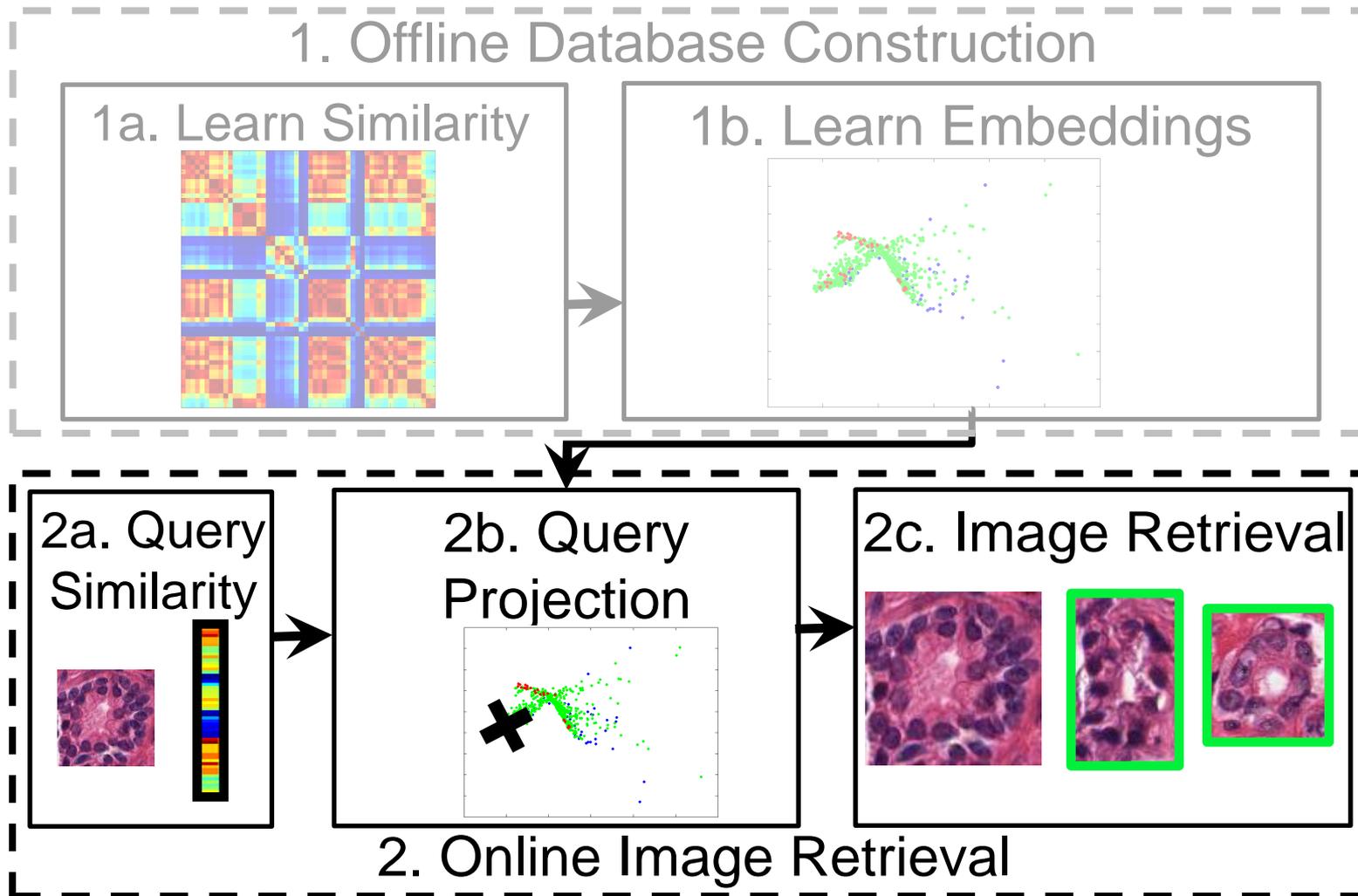
2. Online Image Retrieval

Offline Database Construction

- Step 1: Calculate W for all image in the database
- Step 2: Calculated embeddings y via GE

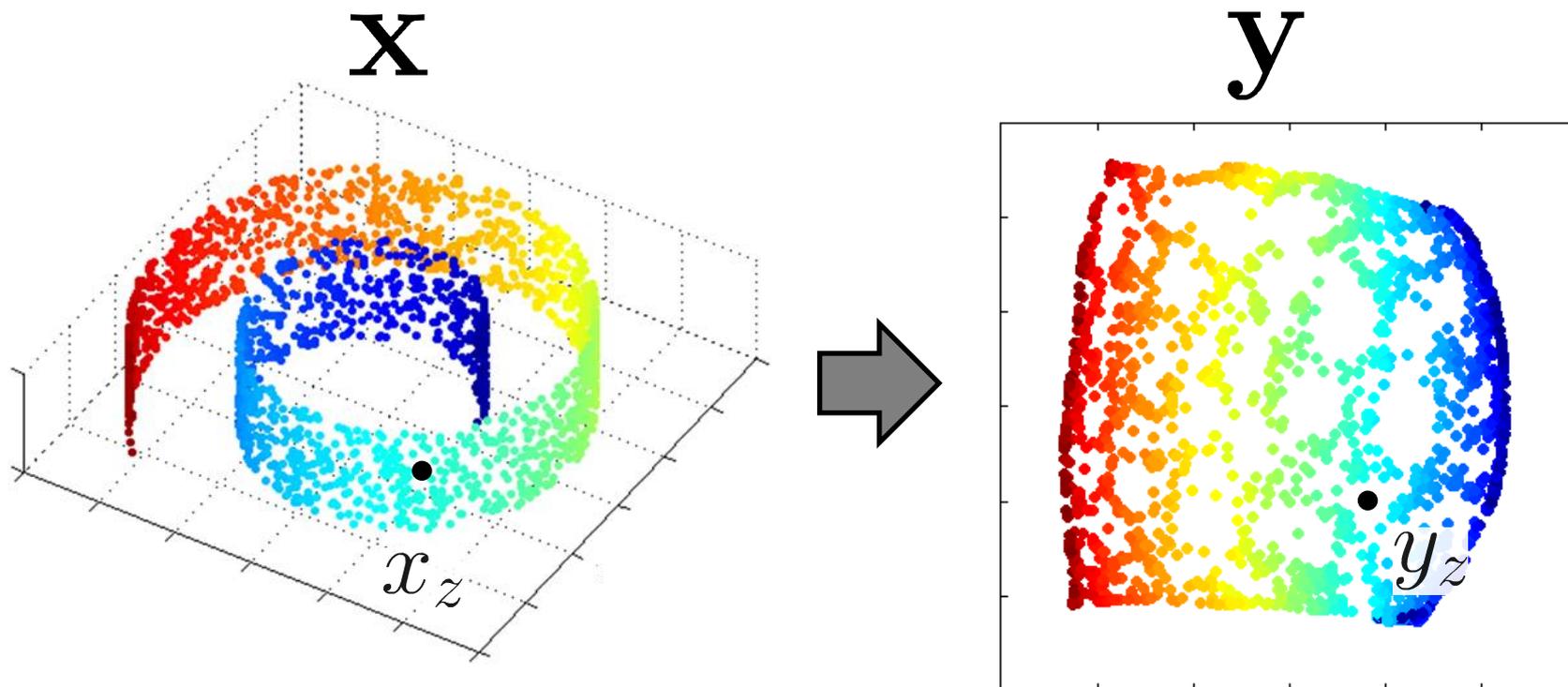


CBIR System



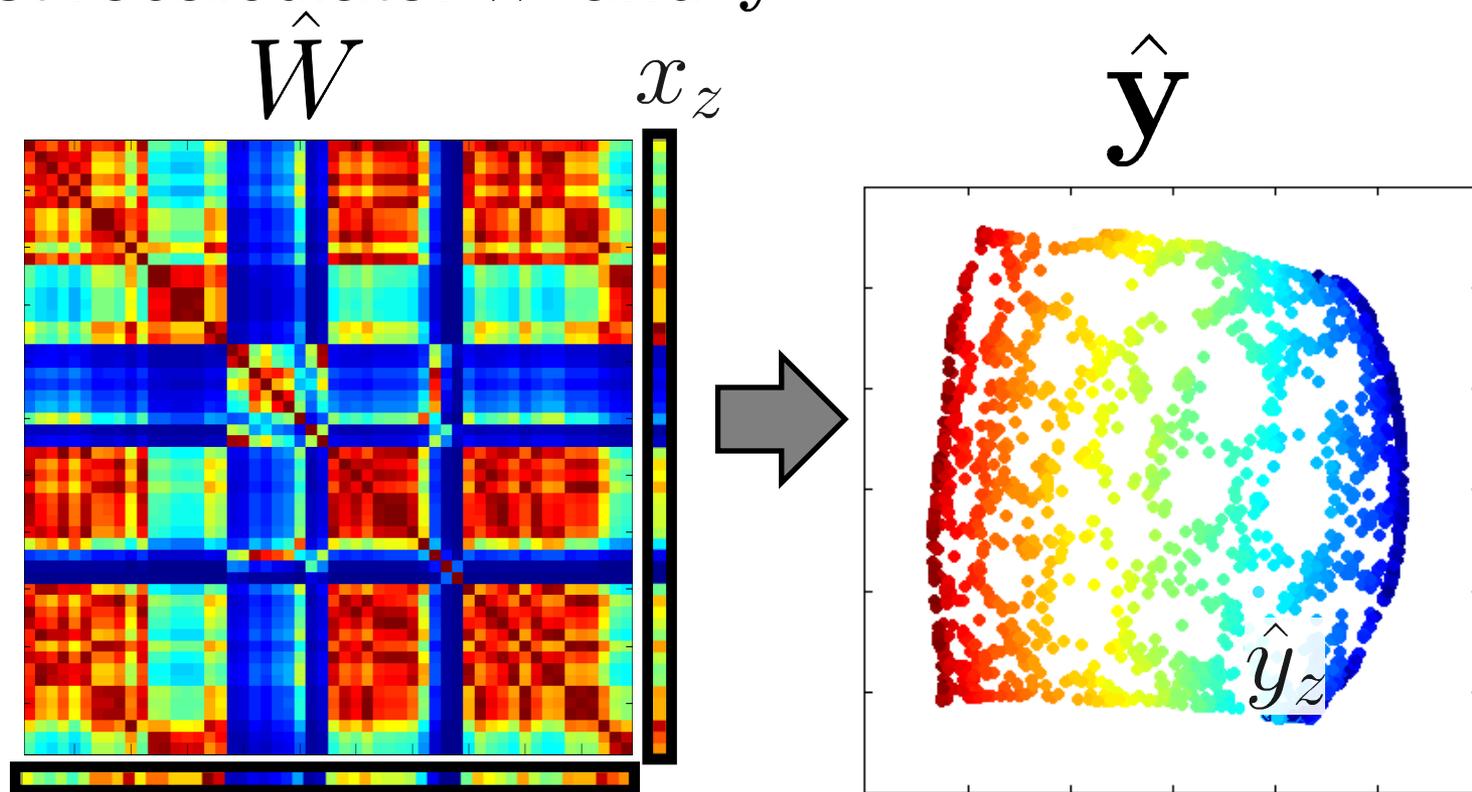
Out-of-Sample Query Image

- Out-of-Sample: query image not in database
- Embedding location unknown



Out-of-Sample Query Image

- Out-of-Sample: query image not in database
- Must recalculate: W and y



Out-of-Sample Query Image

- Out-of-Sample: query image not in database
- Must recalculate: \hat{W} and \hat{y} for every query image
- Requires computationally expensive EVD to find \hat{y}
 - standard EVD run in $O(n^3)$
- Manifold Learning without EVD?

Out-of-Sample Query Image

- Many methods use EVD:
 - Graph Embedding/Laplacian Eigenmaps
 - Local linear embedding (LLE)
 - Isomaps
- Need solutions to deal with out-of-sample images

Locality Preserving Projections (LPP)

- Embeddings estimated by linear combination of the original high dimensional space (He 2003).
- Set $\mathbf{y} = \alpha^T \mathbf{x}$
- LPP formulation:

$$\alpha^T \mathbf{x} = \underset{\mathbf{y}}{\operatorname{argmin}} \sum_{a=1}^N \sum_{b=1}^N \|\alpha^T x_a - \alpha^T x_b\|^2 W(x_a, x_b)$$

Locality Preserving Projections (LPP)

- Embeddings estimated by linear combination of the original high dimensional space (He 2003).
- Set $\mathbf{y} = \alpha^T \mathbf{x}$
- LPP formulation:

$$\mathbf{x}(D - W)\mathbf{x}^T \alpha = \lambda \mathbf{x}D\mathbf{x}^T \alpha$$

Locality Preserving Projections (LPP)

- Embeddings estimated by linear combination of the original high dimensional space (He 2003).

- LPP formulation:

$$\mathbf{x}(D - W)\mathbf{x}^T \alpha = \lambda \mathbf{x}D\mathbf{x}^T \alpha$$

- Original Graph Embedding formulation:

$$(D - W)\mathbf{y}^T = \lambda D\mathbf{y}^T$$

Locality Preserving Projections (LPP)

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- LPP formulation:

$$\mathbf{x}(D - W)\mathbf{x}^T \alpha = \lambda \mathbf{x}D\mathbf{x}^T \alpha$$

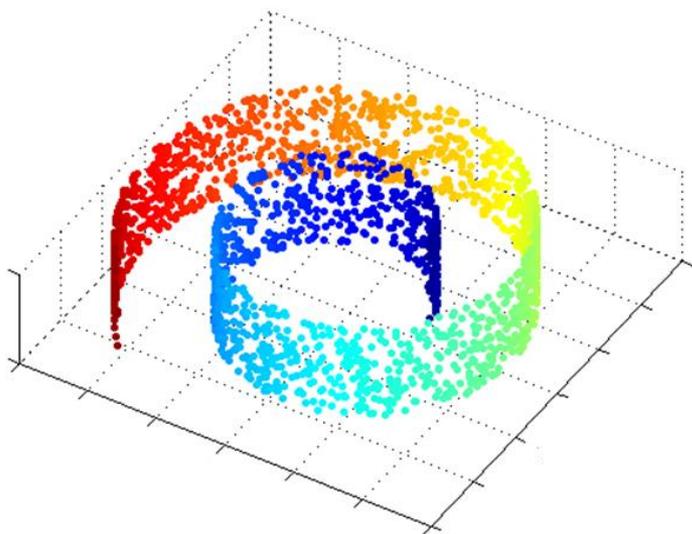
- A new embedding $\hat{y}_z = \alpha^T x_z$

Locality Preserving Projections (LPP)

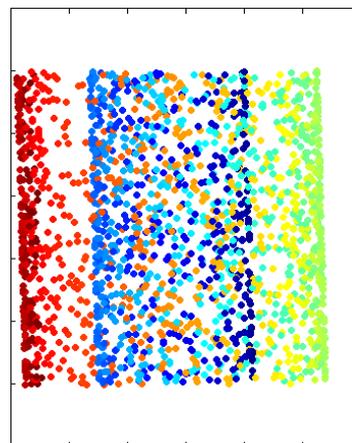
- $\mathbf{y} = \alpha^T \mathbf{x}$ assumes y approximately modeled by a linear combination of \mathbf{x}
- Accurate face recognition (He 2005)

Locality Preserving Projections (LPP)

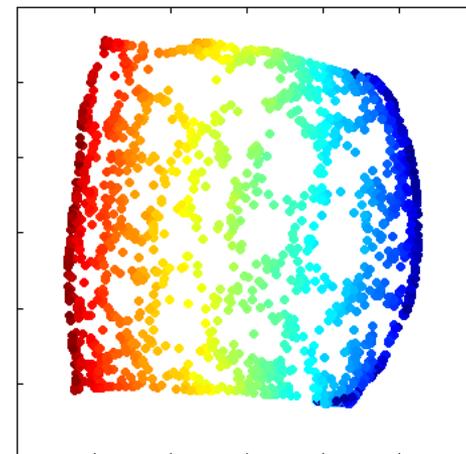
- $\mathbf{y} = \alpha^T \mathbf{x}$ assumes y approximately modeled by a linear combination of \mathbf{x}
- Unable to capture Swiss Roll variation



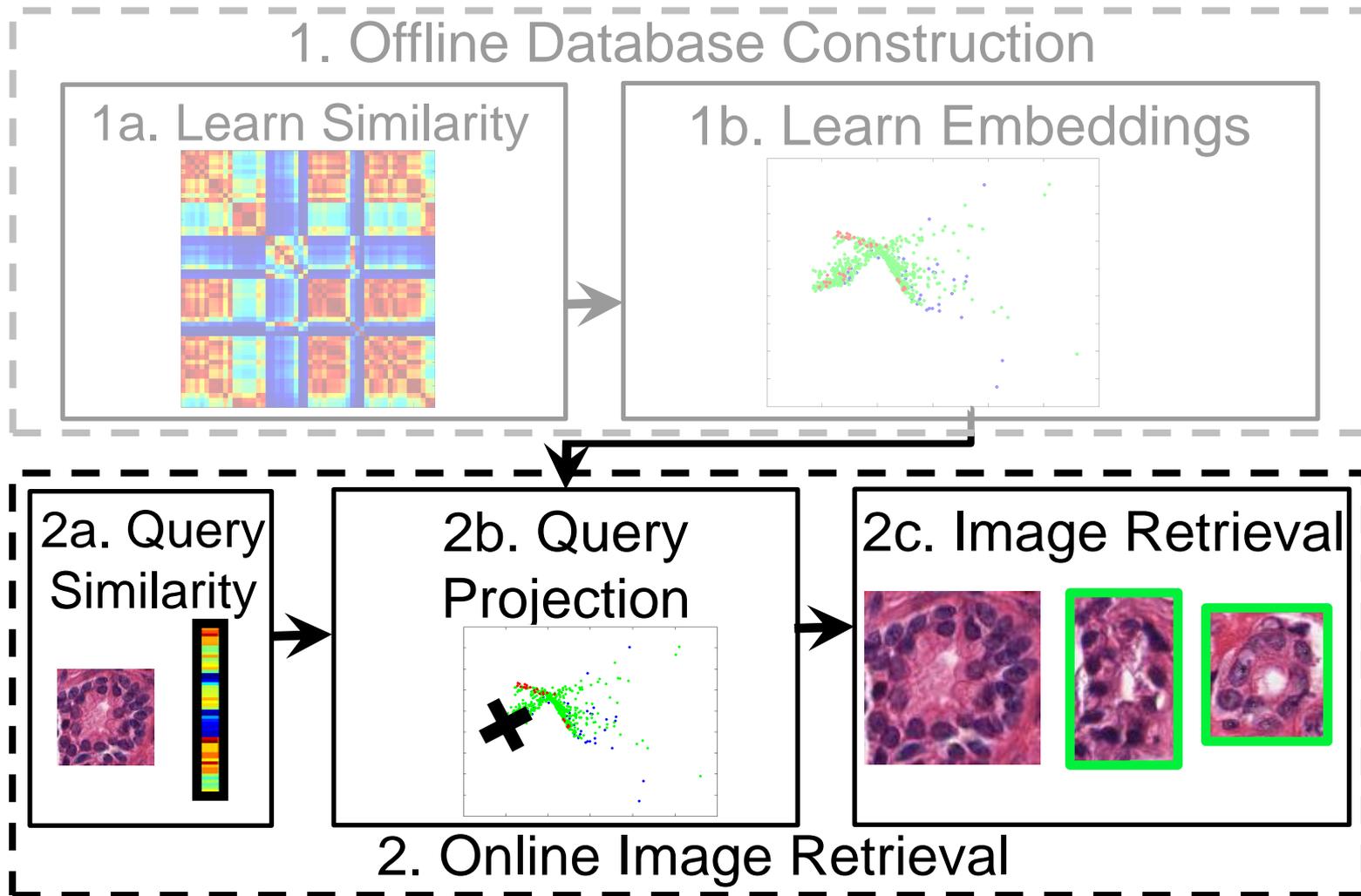
LPP



GE



CBIR System



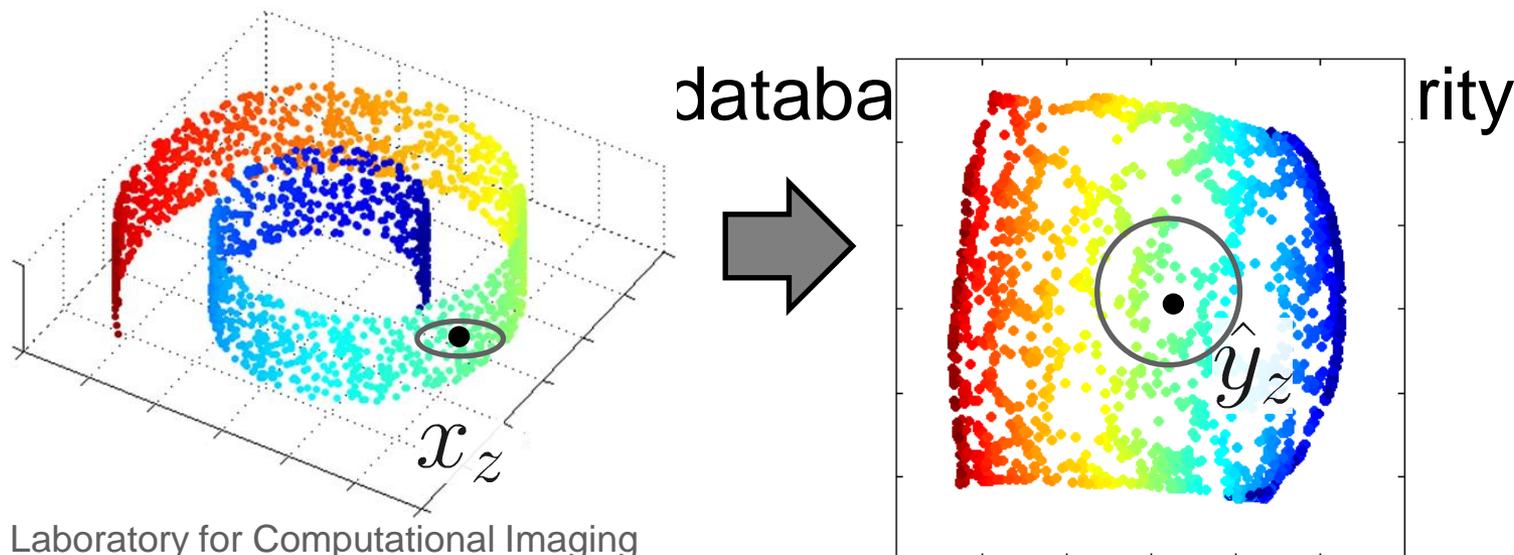
Out-of-Sample Extrapolation (OSE)

- Embeddings are estimated by a linear combination of known embeddings
 - Image segmentation (Fowlkes 2004)
 - CBIR Application (Sparks 2011)

Out-of-Sample Extrapolation (OSE)

- Embeddings are estimated by a linear combination of known embeddings

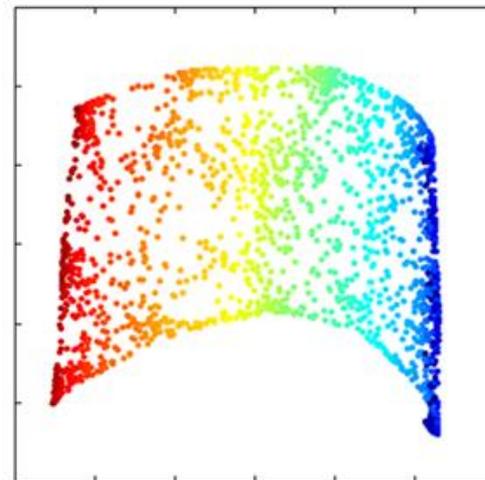
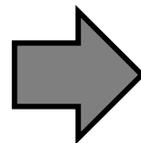
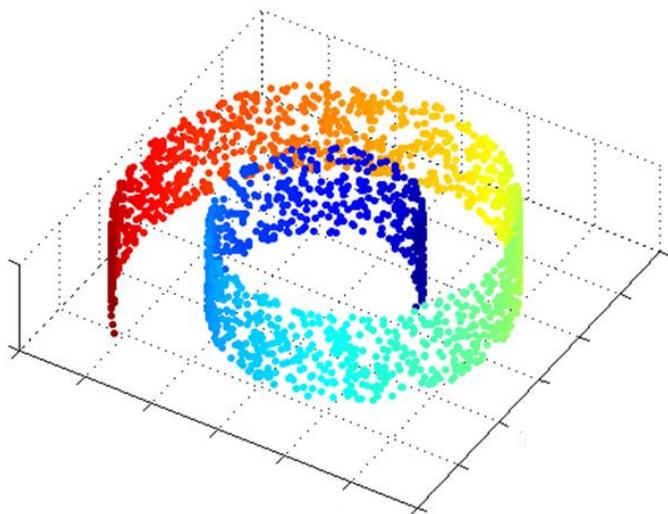
$$\hat{y}_z = \frac{1}{\lambda} \sum_{j=1}^M y_j W(j, z)$$



Out-of-Sample Extrapolation (OSE)

- Large database to learn embedding space

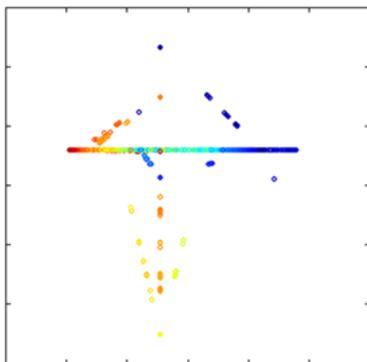
2000 Samples



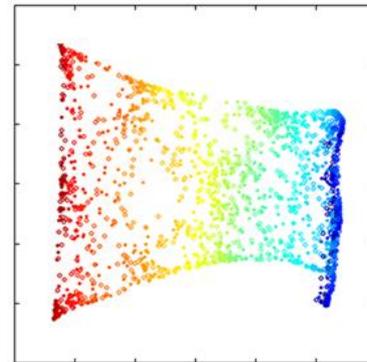
Out-of-Sample Extrapolation (OSE)

- Large database needed to learn embedding space

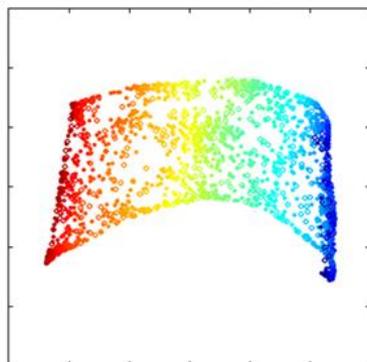
200 Samples



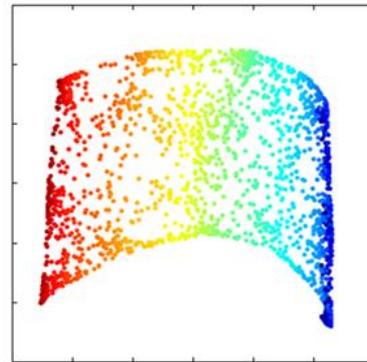
1000 Samples



1400 Samples

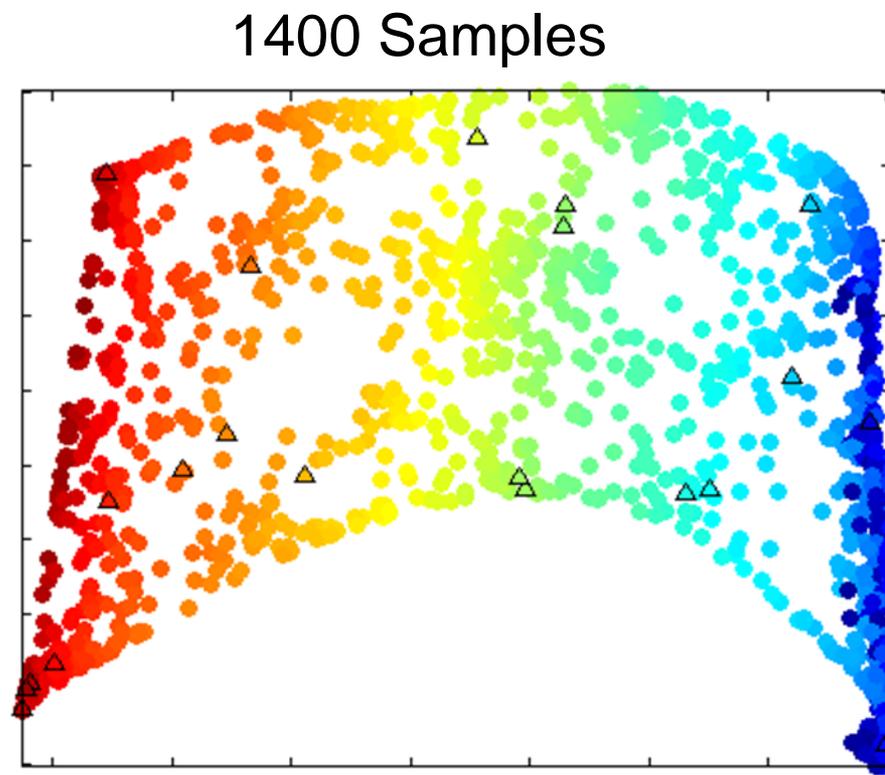


2000 Samples



Out-of-Sample Extrapolation (OSE)

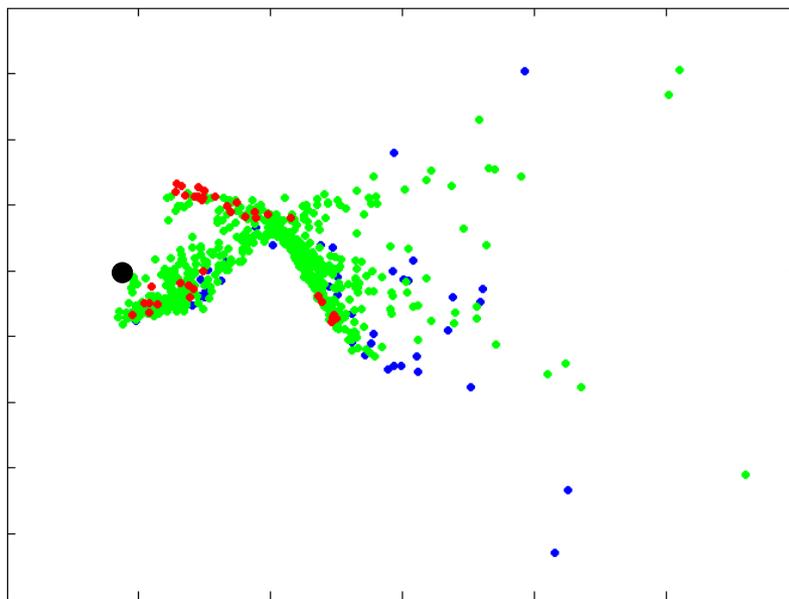
- Large database needed to learn embedding space



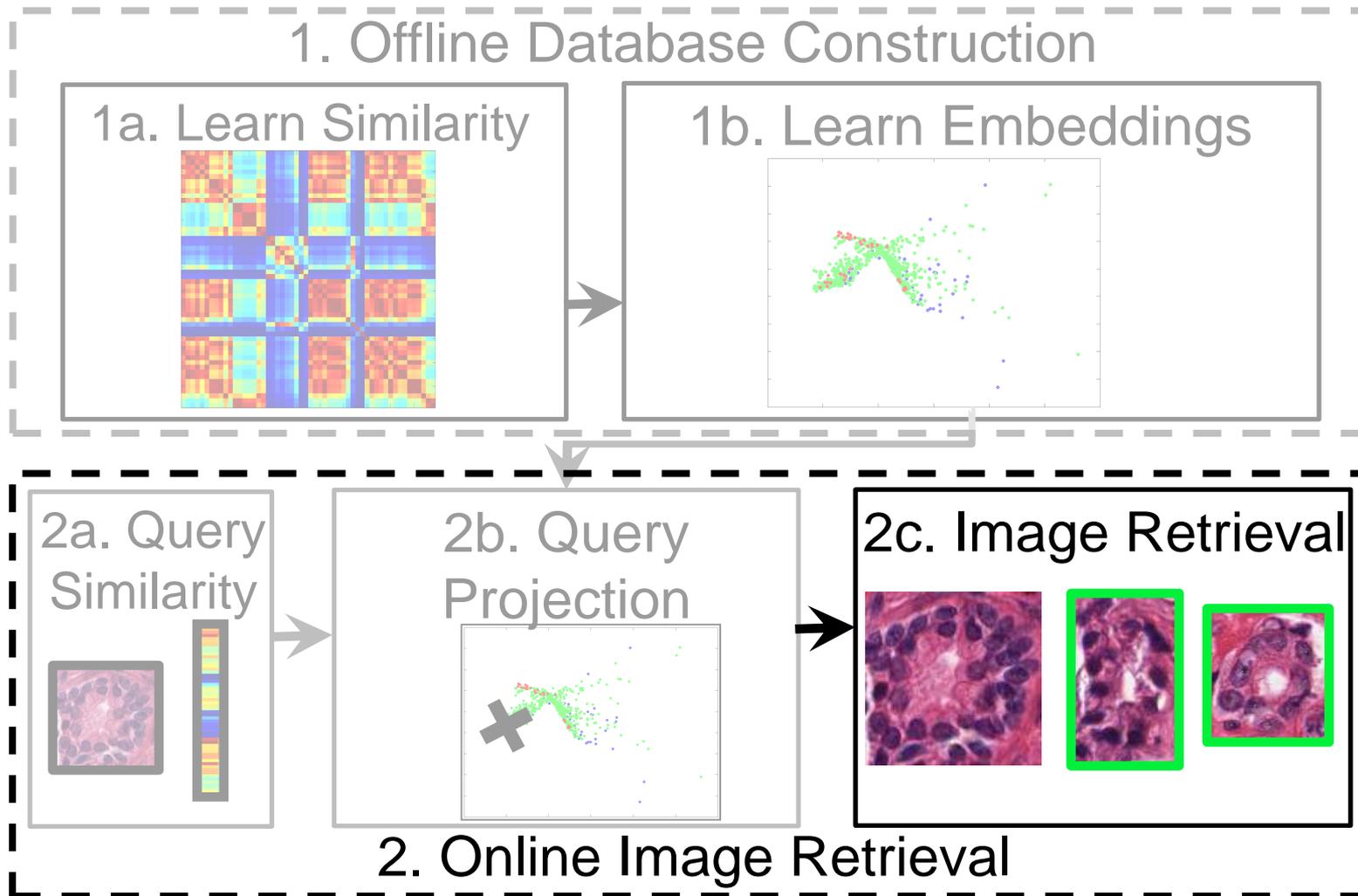
Out-of-Sample Query Image

Overcome out-of-sample query image

- LPP or OSE to efficiently project new query image

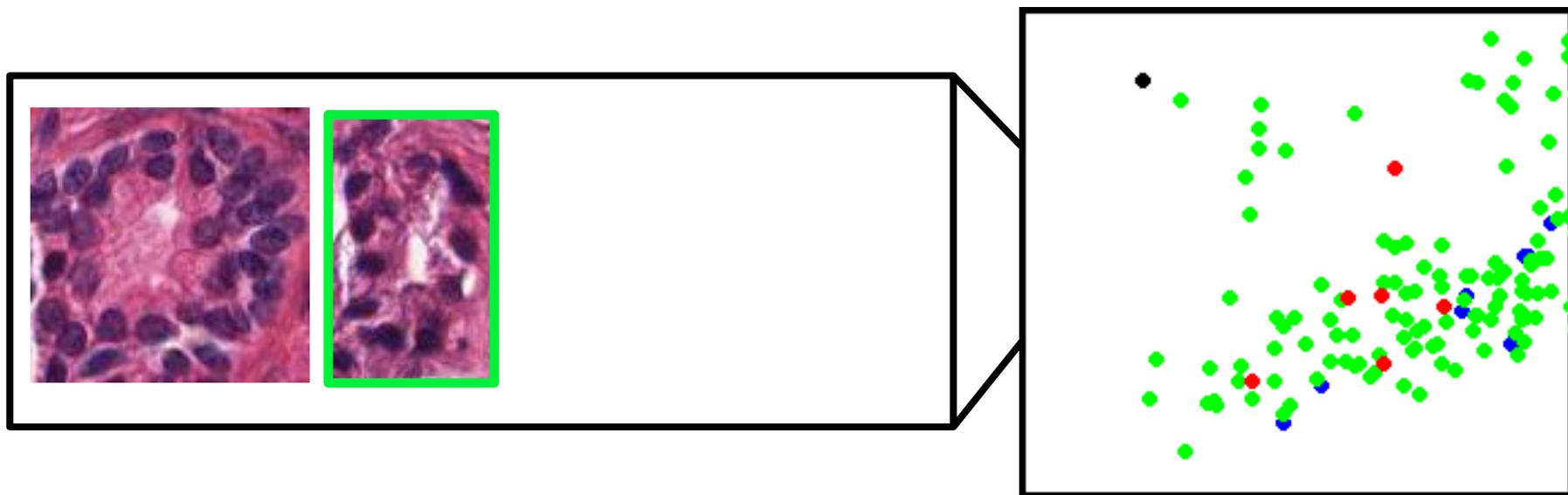


CBIR System



Content-Based Image Retrieval

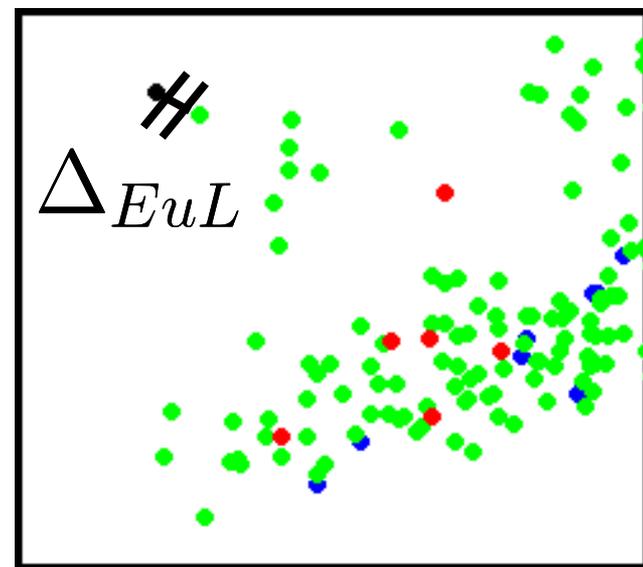
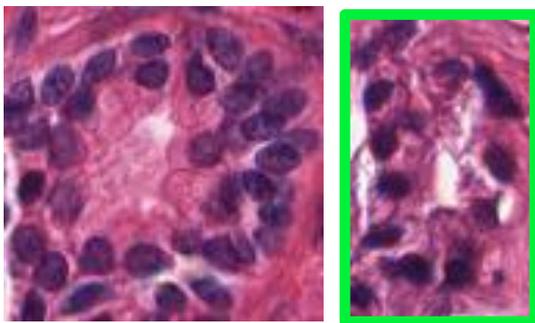
- Embedding location to determine most similar images to query



Content-Based Image Retrieval

- Embedding location to determine most similar images to query

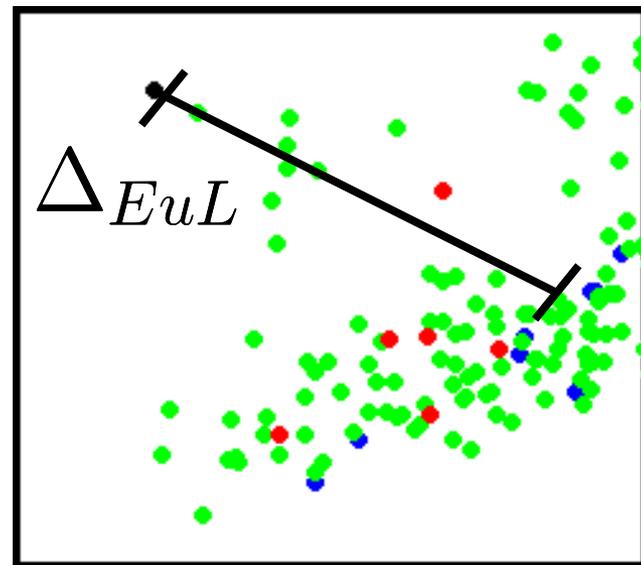
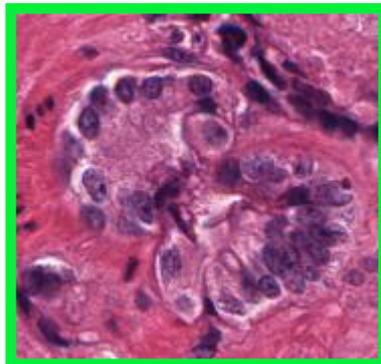
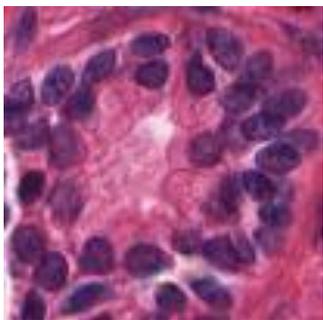
$$\Delta_{EuL}(a, b) = ||y_a - y_b||$$



Content-Based Image Retrieval

- Embedding location to determine most similar images to query

$$\Delta_{EuL}(a, b) = ||y_a - y_b||$$



Key Concepts

1. Preliminaries: Manifold Learning Review

2. Constructing Shape-based Manifolds

Solution: Shape-based similarity matrix

3. Manifold Learning for Content-based Image Retrieval (CBIR)

Solution : LPP or OSE to project query image

Solution: Retrieve by distance in embedding space

Concluding Remarks

- Manifold learning can aid in determining image similarity in a CBIR context
- Out-of-sample images must be taken into account when designing a CBIR system
 - Need to rapidly estimate new embeddings
- Shape-based similarity measures to retrieval images according to object similarity

Thanks to our funding agencies

- Wallace H. Coulter Foundation
- New Jersey Commission on Cancer Research
- National Cancer Institute (Grant Nos. R01CA136535-01 and R01CA14077201)
- The Cancer Institute of New Jersey
- Department of Defense Prostate Cancer Research Program (W81XWH-10-1-986)