

Algorithm outline:

- Compute fiducial location error (FLE) for every fiducial
- Rotate fiducial location error (FLE) into target coordinate system
- Compute marker target error (MTE) at the centroid in target coordinates
- Compute target registration error (TRE) at a point of interest in target coordinates
- Compute application-specific matrix norms

Estimation of the FLE from given IPE:

- Assume that all errors are covered in some Gaussian unbiased noise on the image planes
- Use error propagation based only on the geometric constellation of the cameras

From p :

$$\vec{u}_1 = \rho_1 K_1 T_1 \vec{x}$$

$$\vdots$$

$$\vec{u}_n = \rho_n K_n T_n \vec{x}$$

compute $\Sigma_{\vec{c}} = \left(M^T \begin{bmatrix} R^T \Sigma_{p_1} R & 0 \\ \vdots & \vdots \\ 0 & R^T \Sigma_{p_n} R \end{bmatrix}^{-1} M \right)^+$

- C++ code analytically generated with Mathematica

Estimation of the MTE from given FLE

- Marker Target Error (MTE): Error in the centroid of the target
- Use 3D/3D pose estimation algorithm to get $R\vec{q}_k + \vec{t} = p_k + \Delta\vec{p}_k$
- Treat c as a random variable representing the error and build

$$J_f(\vec{q}) = \frac{\partial f(\vec{p}, \vec{q})}{\partial \vec{c}} \Big|_{\vec{c}=\vec{0}} = \begin{bmatrix} 1 & 0 & 0 & 0 & q_z - q_y \\ 0 & 1 & 0 & -q_z & 0 & q_x \\ 0 & 0 & 1 & q_y - q_x & 0 & 0 \end{bmatrix}$$

with $f(\vec{p}, \vec{q}) = R\vec{q} + \vec{t} - \vec{p}$

This maps the 6D MTE to the 3D FLEs $\begin{bmatrix} \Delta p_1 \\ \vdots \\ \Delta p_n \end{bmatrix} = \begin{bmatrix} J_f(q_1) \\ \vdots \\ J_f(q_n) \end{bmatrix} \Delta x = M \Delta x$

- Using the backward propagation formula, we get

$$\Sigma_{\vec{c}} = \left(M^T \begin{bmatrix} R^T \Sigma_{p_1} R & 0 \\ \vdots & \vdots \\ 0 & R^T \Sigma_{p_n} R \end{bmatrix}^{-1} M \right)^+$$

where $R^T \Sigma_{p_n} R$ is the FLE in the target coordinate system

- MTE is given in the target coordinate system

Estimation of the TRE from given MTE

- MTE is given at the centroid, needs forward propagation to get the error at some point of interest
- Use again

$$J_f(q) = \begin{bmatrix} 1 & 0 & 0 & 0 & q_z - q_y \\ 0 & 1 & 0 & -q_z & 0 & q_x \\ 0 & 0 & 1 & q_y - q_x & 0 & 0 \end{bmatrix}$$

and compute

$$\Sigma_{\vec{p}} = J_f \Sigma_{\vec{c}} J_f^T$$

- Resulting error at the point of interest is the sum of the translational part in the MTE at the centroid and the propagated angular part in the MTE

Experimental Estimation of IPE

- Small baseline: 20 cm
- 4x4 grid of fiducial points
- Resulting error magnified by factor 50
- Assuming unbiased Gaussian noise
- Artifacts due to discretization in tracking algorithms

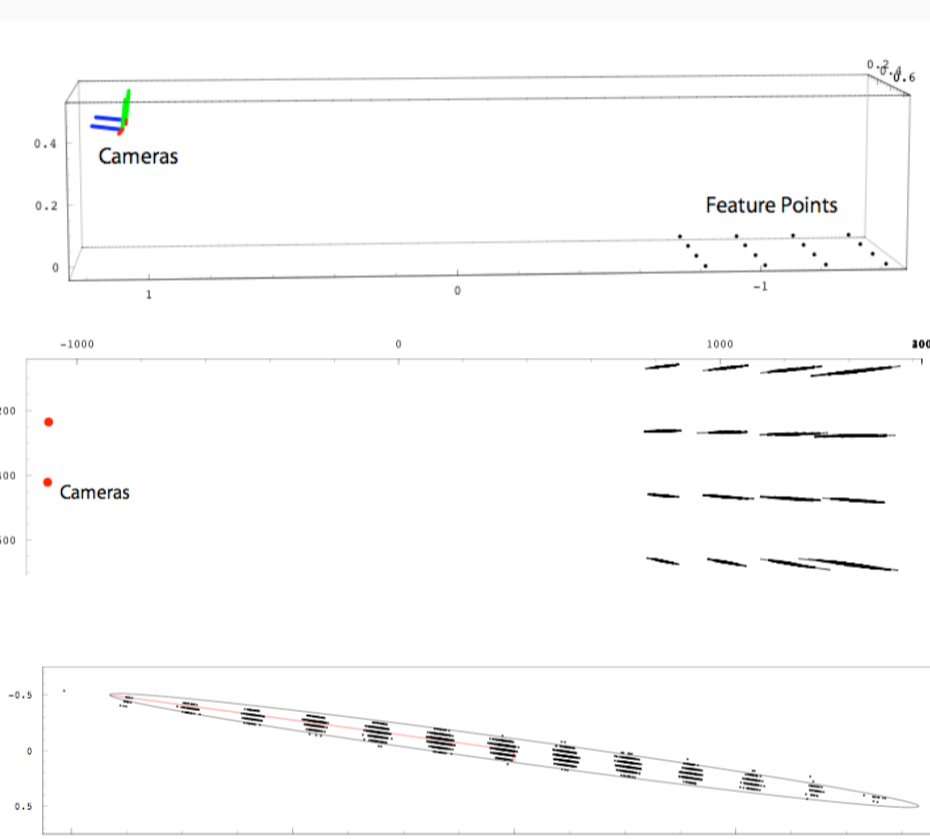
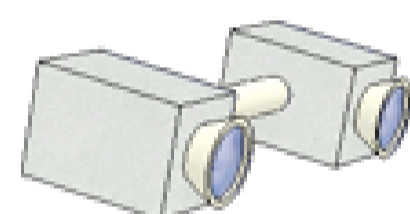
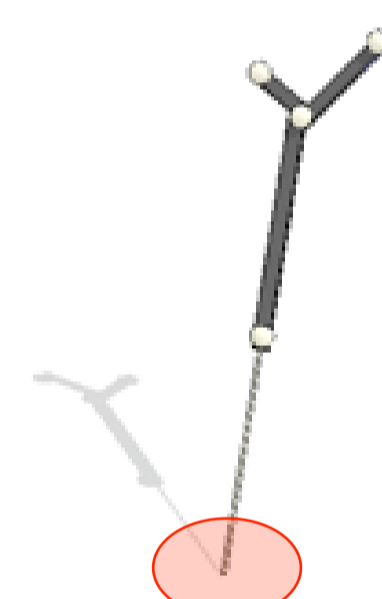


Image plane error (IPE):
Gaussian error on the image plane (2DoF).



Fiducial Location Error (FLE):
How good can we determine the location of the fiducials in space (3DoF)

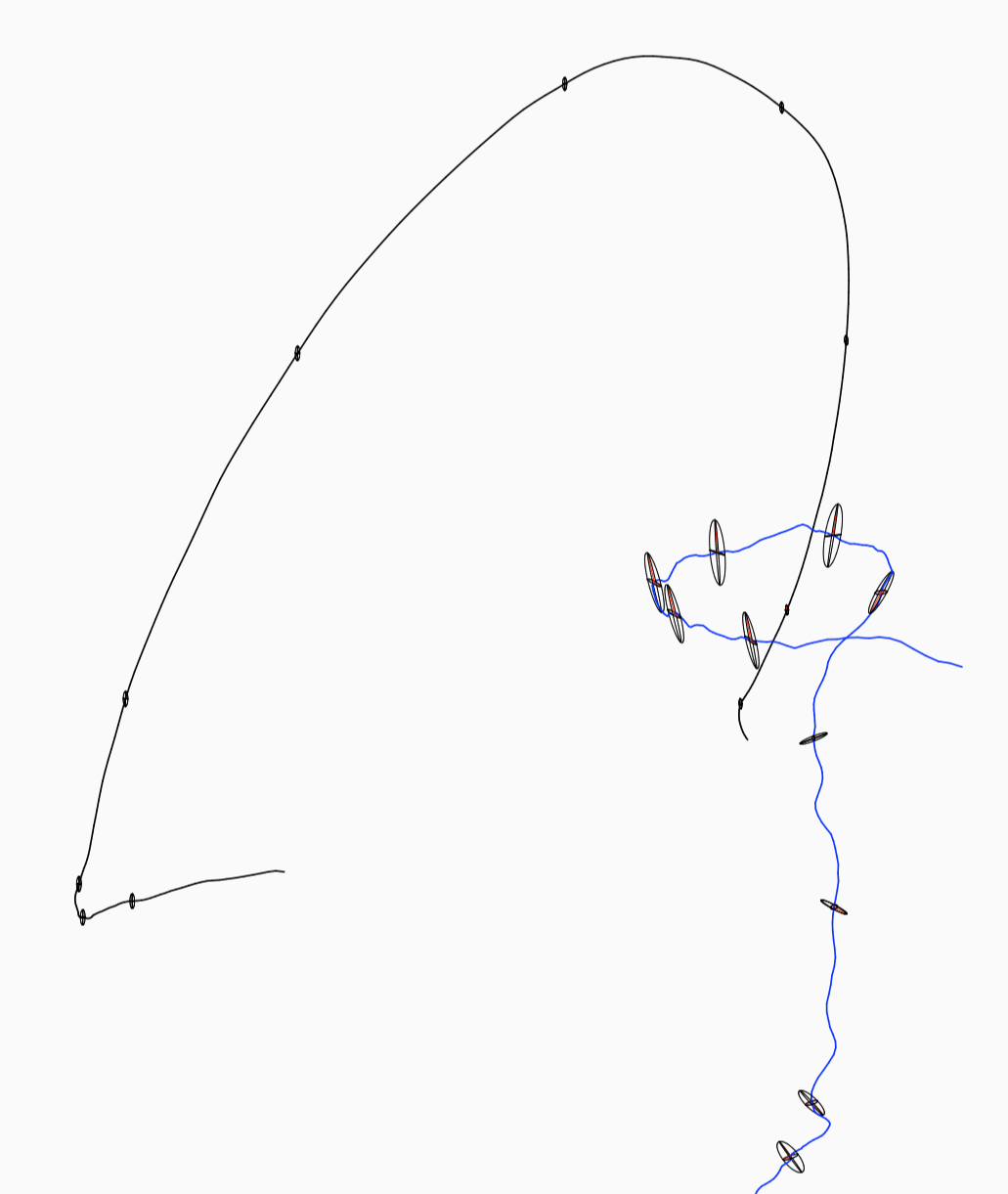
Marker Target Error (MTE):
How good can the target be detected (6DoF)?



Target Registration Error (TRE):
What is the final error at the point of interest (3DoF)?

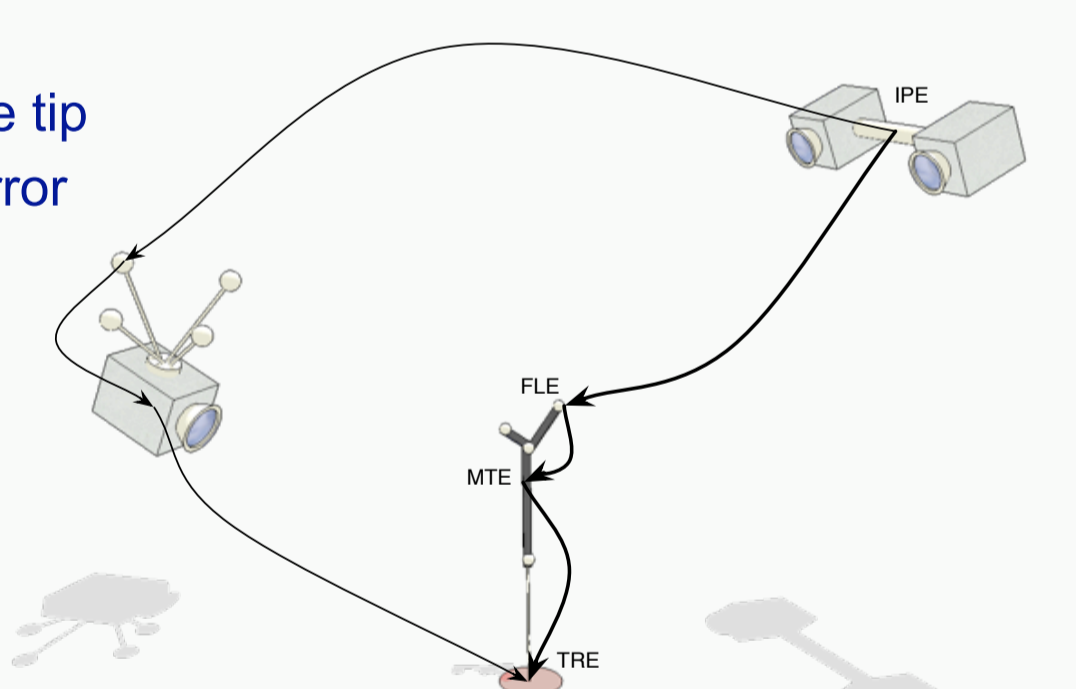
Orientation matters:

- Significant unisotropic behavior in both FLE error and error propagation
- Example path with propagation:
 - Same FLE over whole path
 - Significant differences in the resulting TRE
- Try not to amplify already large errors!

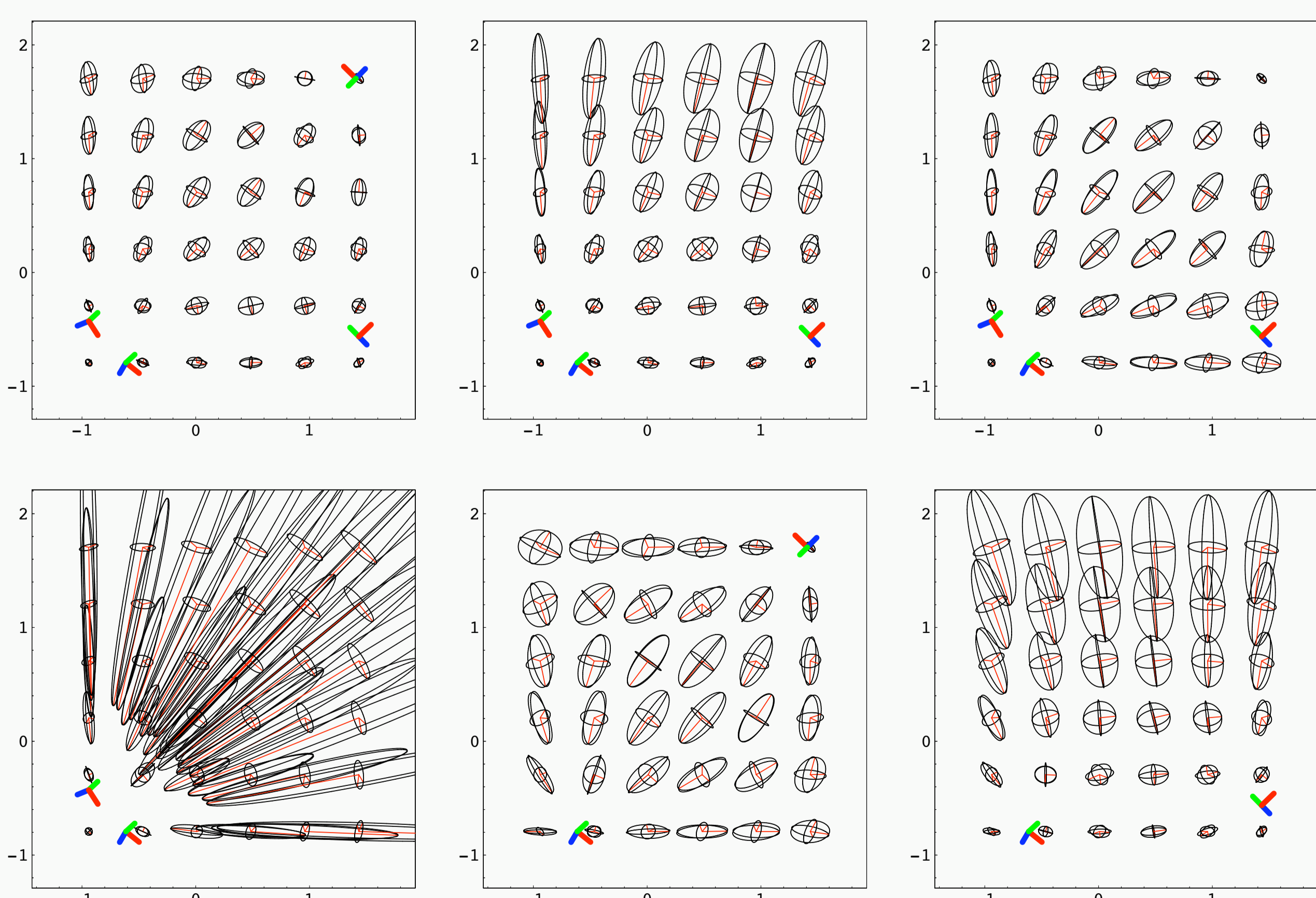


Demo Setup

- This demo covers only the pose estimation error for the tip
- The image augmentation error is neglected here
- Just for visualization of the TRE, not for overall application-specific error



Example Setups



Comparison with the state of the art

- TRE (Fitzpatrick et al. 1998) is de facto standard
- Assume FLE is given
- d_k is distance of the point of interest from the k-th principal axis
- f_k is RMS distance of the fiducials from the k-th principal axis

$$TRE^2 = \frac{FLE^2}{N} \left(1 + \frac{1}{3} \sum_{k=1}^3 \frac{d_k^2}{f_k^2} \right)$$

- How can we estimate the fiducial location error (FLE)?

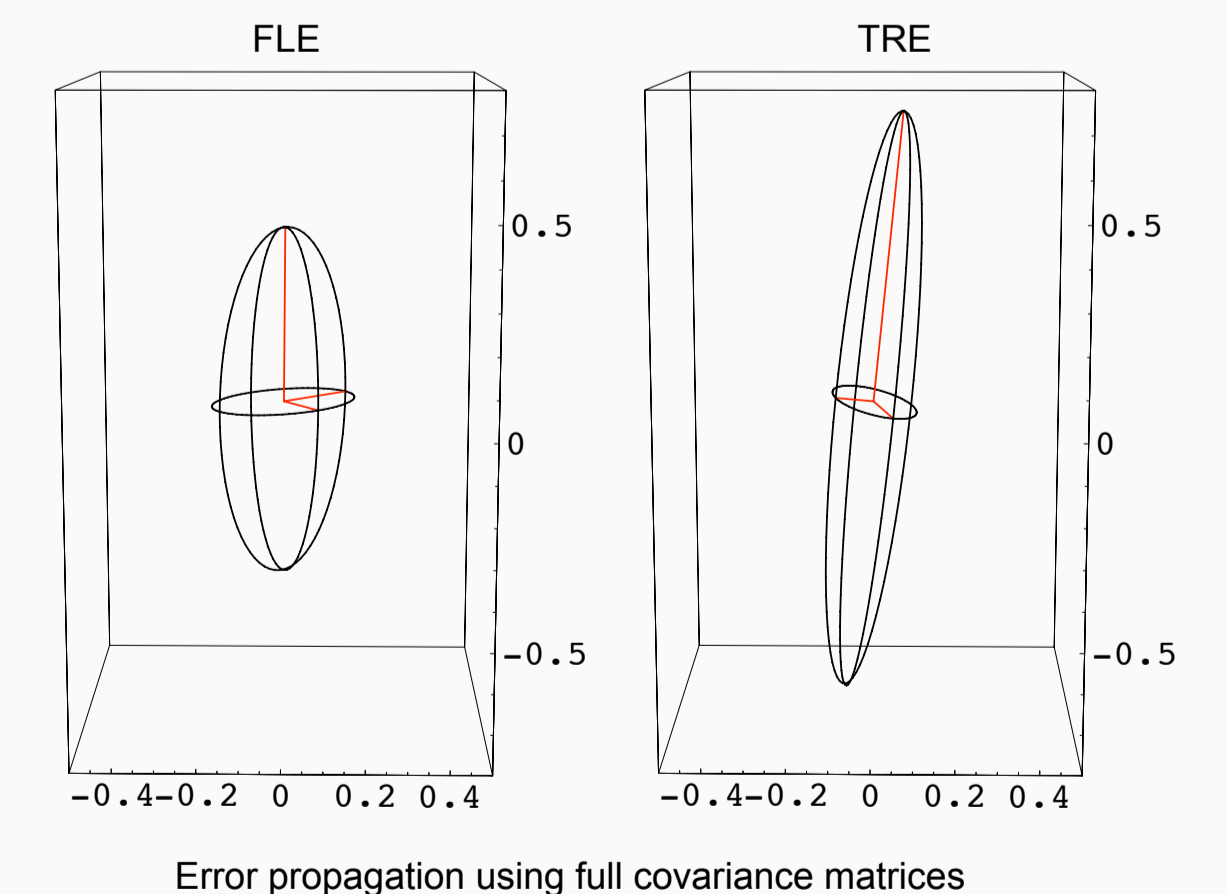
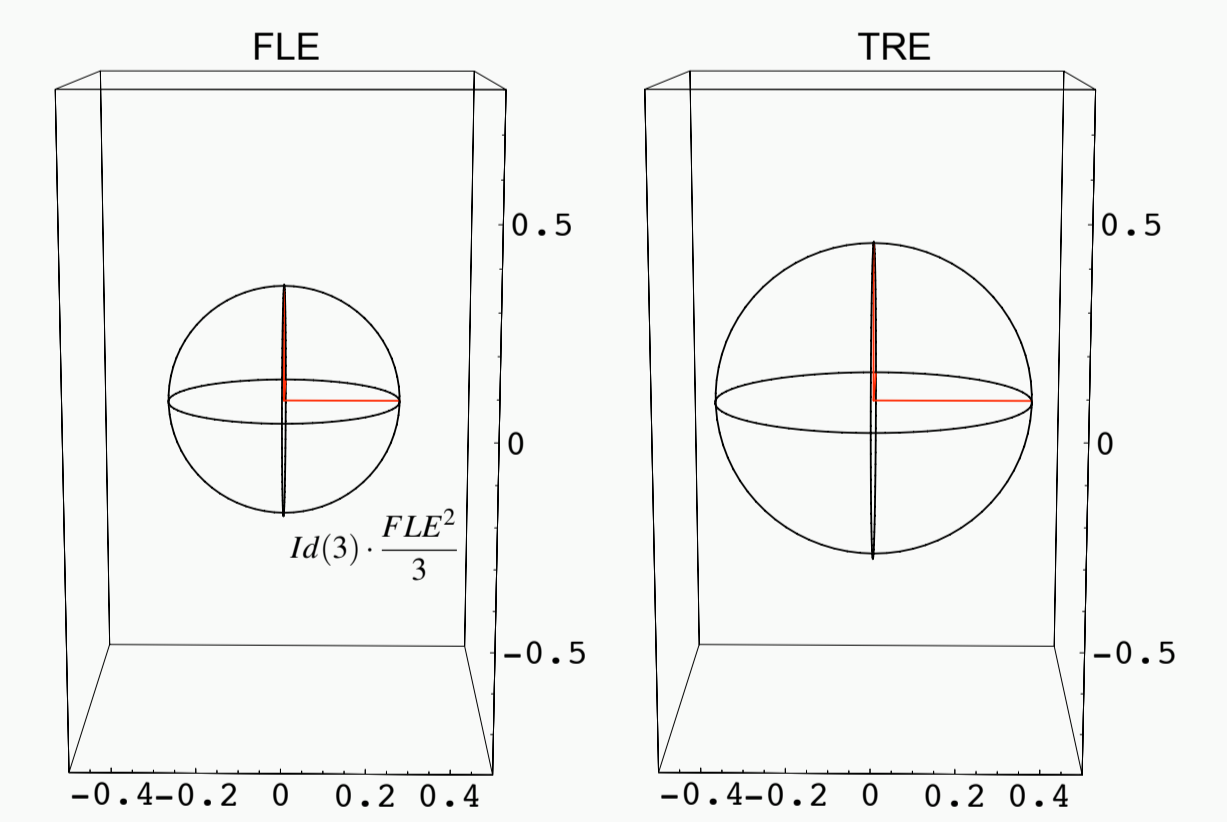
- Experiments
- Data sheets from the tracking system manufacturer
- „Magic numbers“ from the tracking software
- We propose an analytical model

- Are the assumptions correct?

„This application assumes that [...] the fiducial measurement error is identical, independent, zero-mean, and isotropic. [...] Generally FLE is slightly anisotropic, with error along the optical axis of the OPS higher than error perpendicular to this axis.“

- We propose using general covariance matrices

- For comparison, use an FLE error with same RMS, but not assuming independence and isotropy
- Example target: Commercial pointer with four balls, POI about 15 cm from centroid
- Initial RMS: 0.125953 mm, sqrt of eigenvalues: {0.07272} vs. {0.11023, 0.05083, 0.03363}
- Resulting RMS at the tip: 0.172376 mm with sqrt of eigenvalues {0.09952} for the TRE vs. 0.202958 mm with sqrt of eigenvalues {0.182091, 0.0863337, 0.0241093} for the proposed method.



Related Work & References

- W. Hoff and T. Vincent. Analysis of head pose accuracy in augmented reality. In IEEE Transactions on Visualization and Computer Graphics, volume 6(4), pages 319–334, 2000
- M. Fitzpatrick, J. B. West, and C. R. Maurer, Jr. Predicting error in rigid-body, point-based registration. IEEE Transactions on Medical Imaging, 17(5):694–702, 1998.
- B. D. Allen and G. Welch. A general method for comparing the expected performance of tracking and motion capture systems. Proc. of Virtual reality software and technology VRST 2005