

# NURBS Warps

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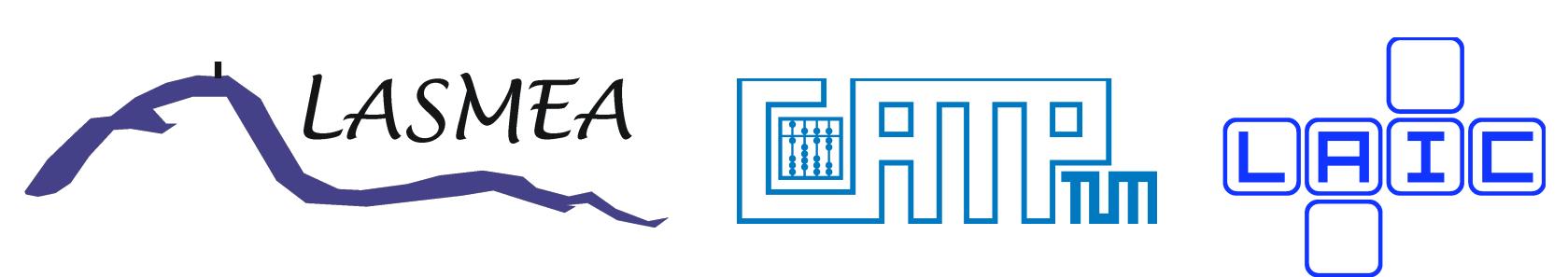
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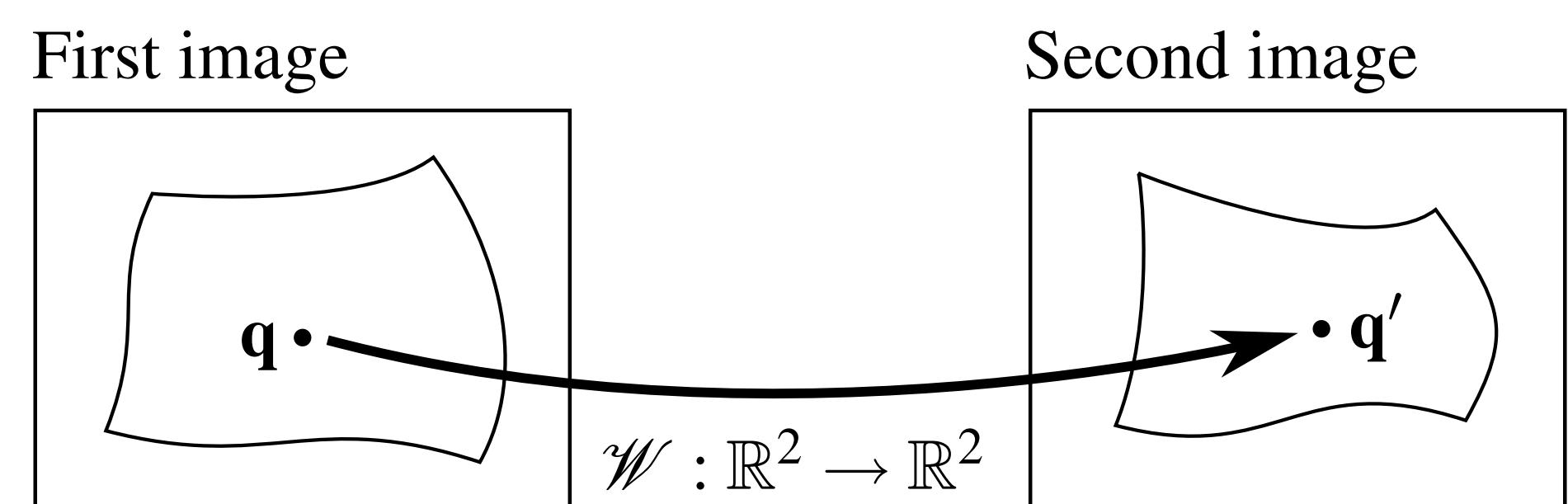
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## Goal / Contribution

- Context: model the **warp** between two images of a deformable surface
- **B-Splines** (or Free Form Deformations) have been proved efficient [1]
- However, B-Splines correspond to **affine** imaging conditions
- We propose a new model: the **Non Uniform Rational B-Spline** (NURBS) warps
- These are an extension of the B-Spline warps but for **perspective** imaging conditions

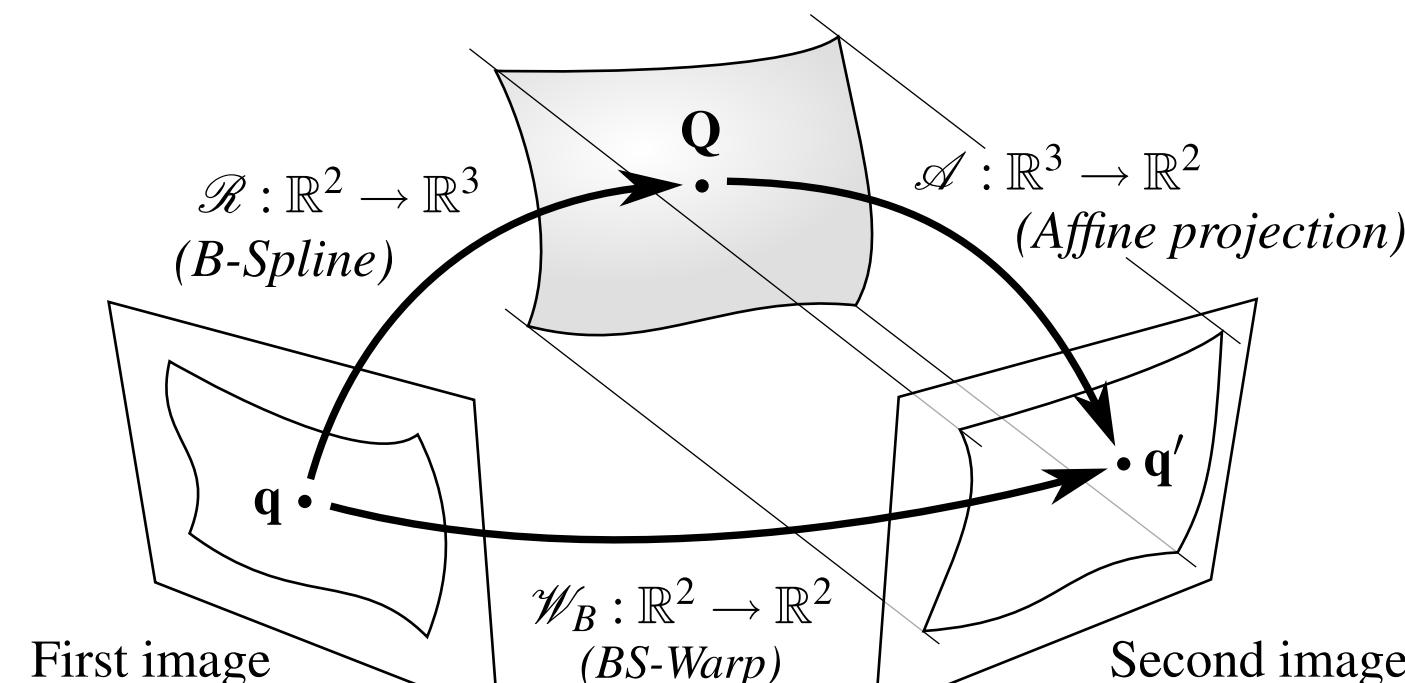


## BS-Warps

B-Spline Warps (Free Form Deformations)

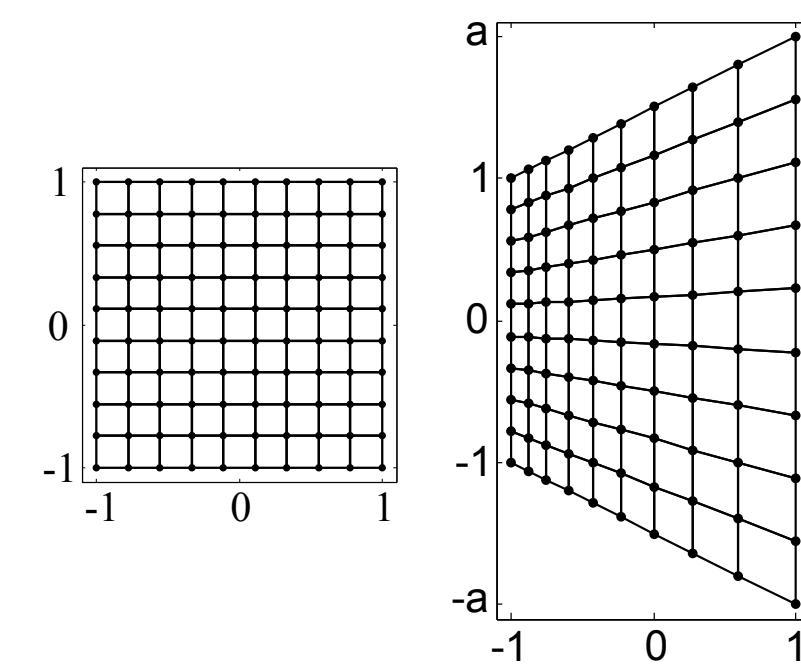
Corresponds to affine imaging conditions

It models poorly the perspective effects



$$\mathbf{q}' = \mathcal{A}(\mathbf{Q}) = \sum_{i=1}^m \sum_{j=1}^n \begin{pmatrix} p_{i,j}^x \\ p_{i,j}^y \\ p_{i,j}^z \end{pmatrix} N_i(x) N_j(y)$$

Affine projection



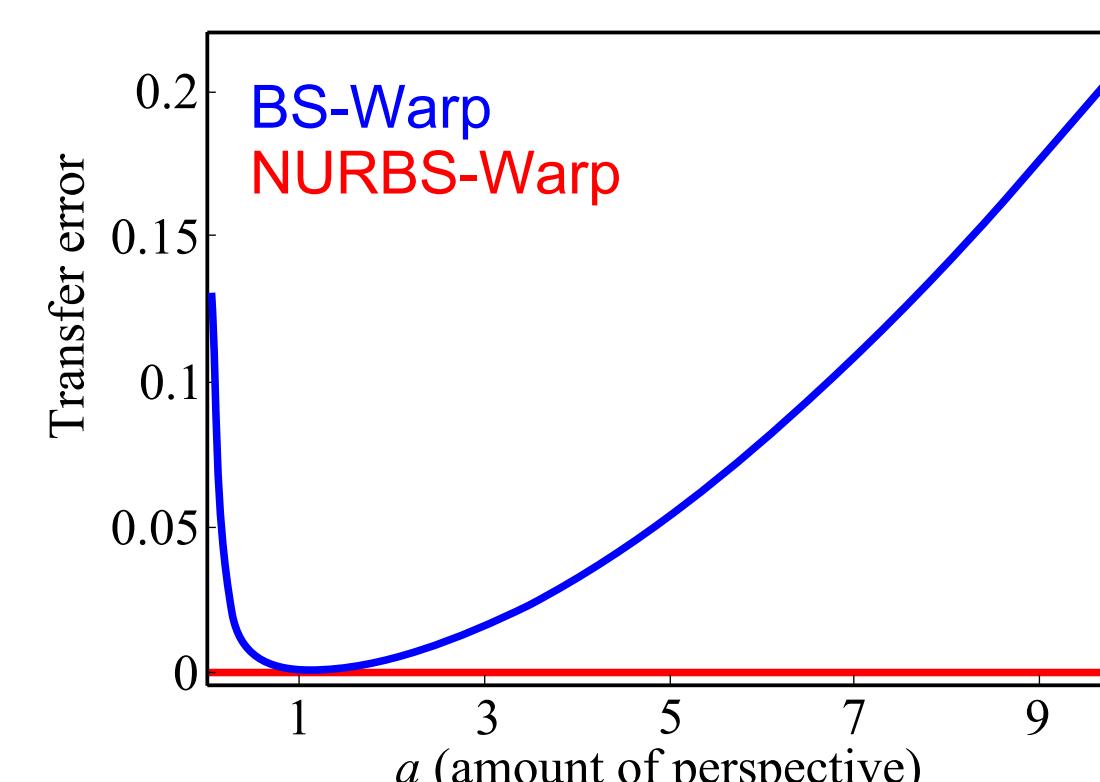
$$\mathbf{q} = (x \ y)^T$$

↓  
'Back-projection'  
2D → 3D

$$\mathbf{Q} = \mathcal{R}(\mathbf{q}; \mathbf{x}) = \sum_{i=1}^m \sum_{j=1}^n \begin{pmatrix} p_{i,j}^x \\ p_{i,j}^y \\ p_{i,j}^z \end{pmatrix} N_i(x) N_j(y)$$

Perspective projection

$$\mathbf{q}' = \mathcal{P}(\mathbf{Q}) = \frac{\sum_{i=1}^m \sum_{j=1}^n \begin{pmatrix} p_{i,j}^x \\ p_{i,j}^y \\ p_{i,j}^z \end{pmatrix} w_{i,j} N_i(x) N_j(y)}{\sum_{i=1}^m \sum_{j=1}^n w_{i,j} N_i(x) N_j(y)}$$

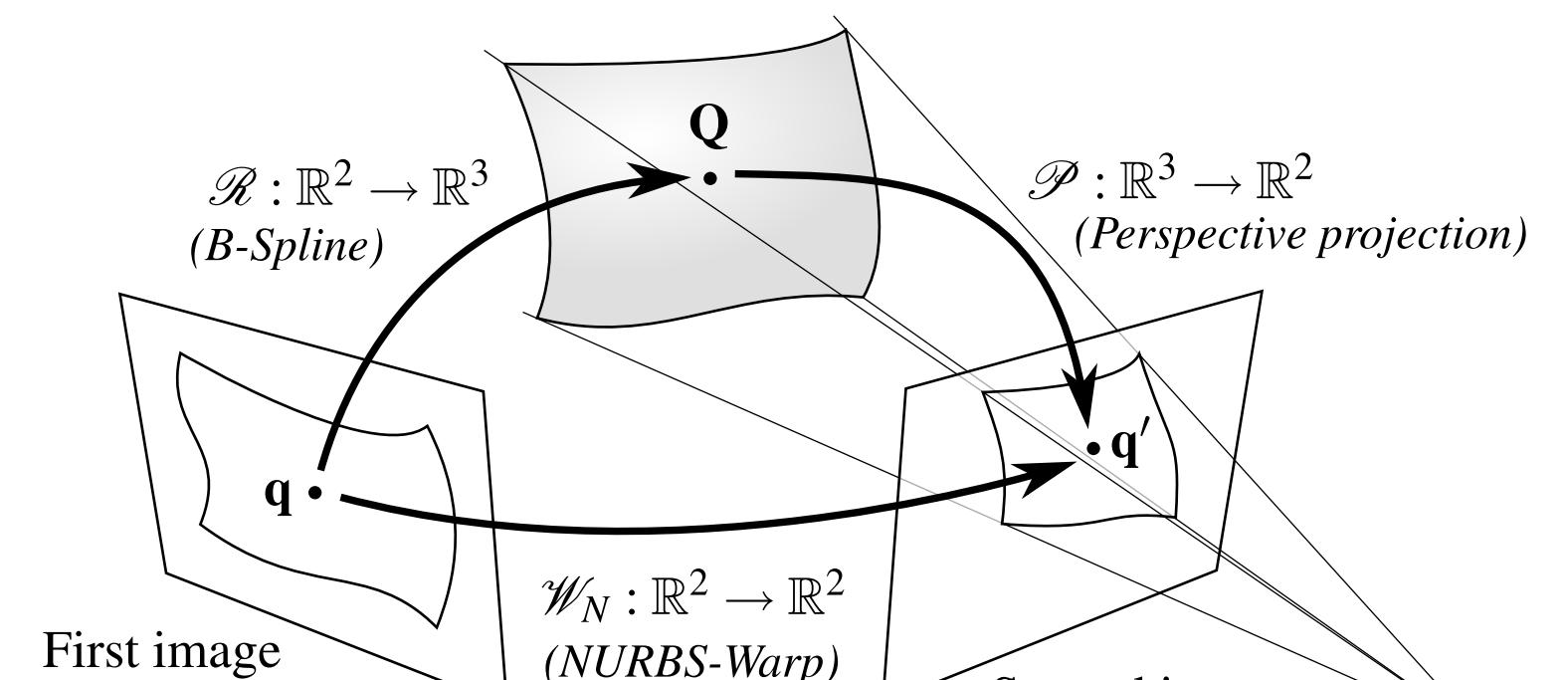


## NURBS-Warps

Non Uniform Rational B-Spline Warps

Corresponds to perspective imaging conditions

It includes the 'division of the perspective'



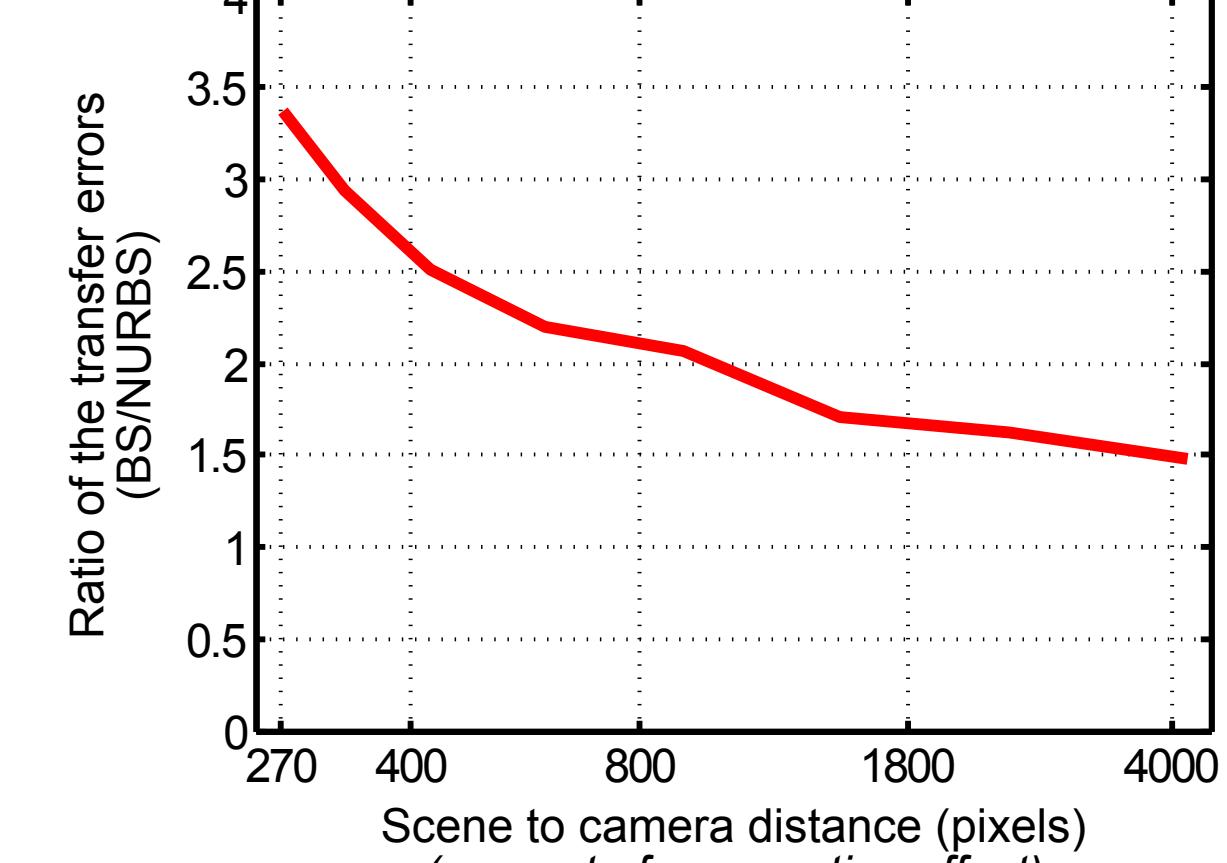
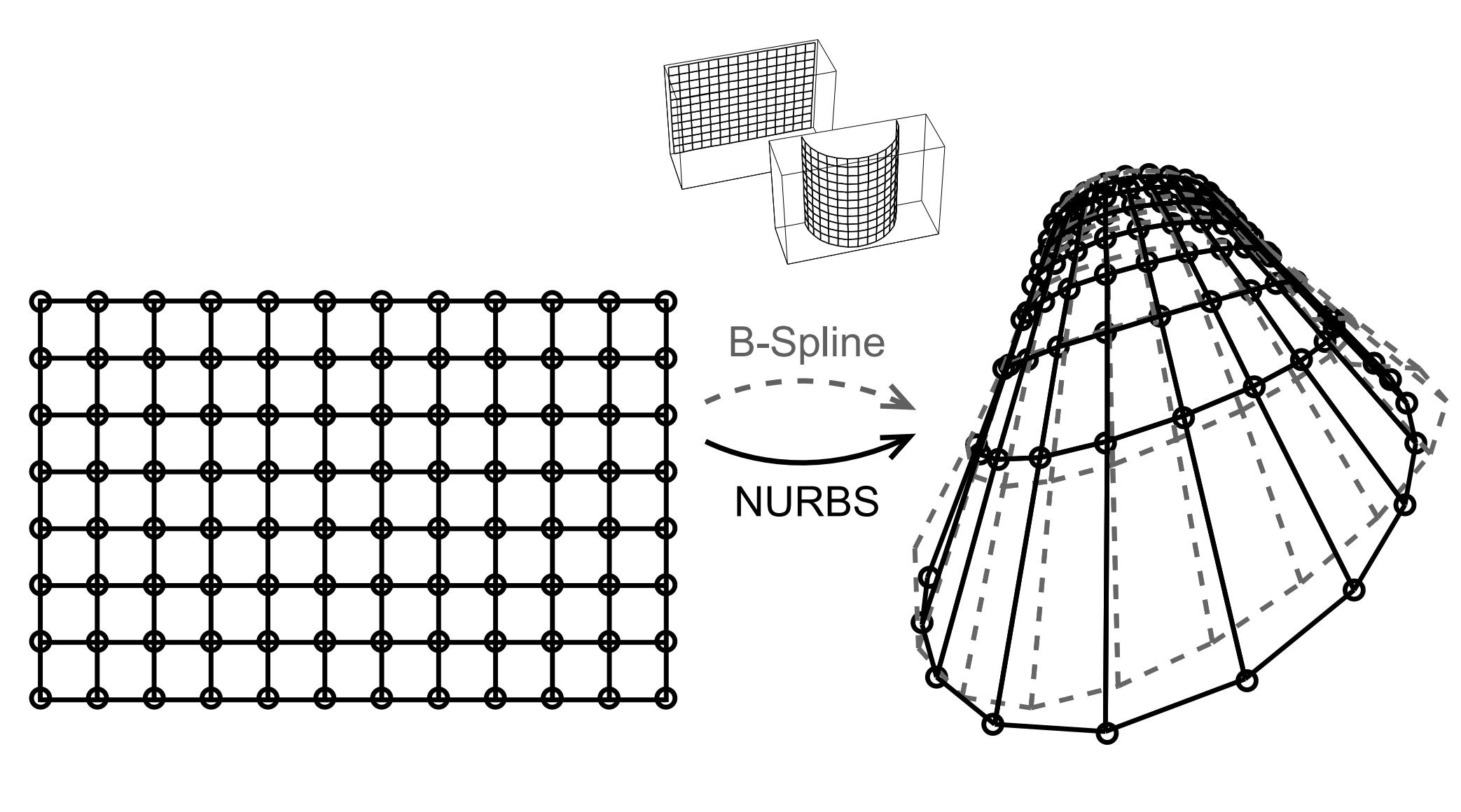
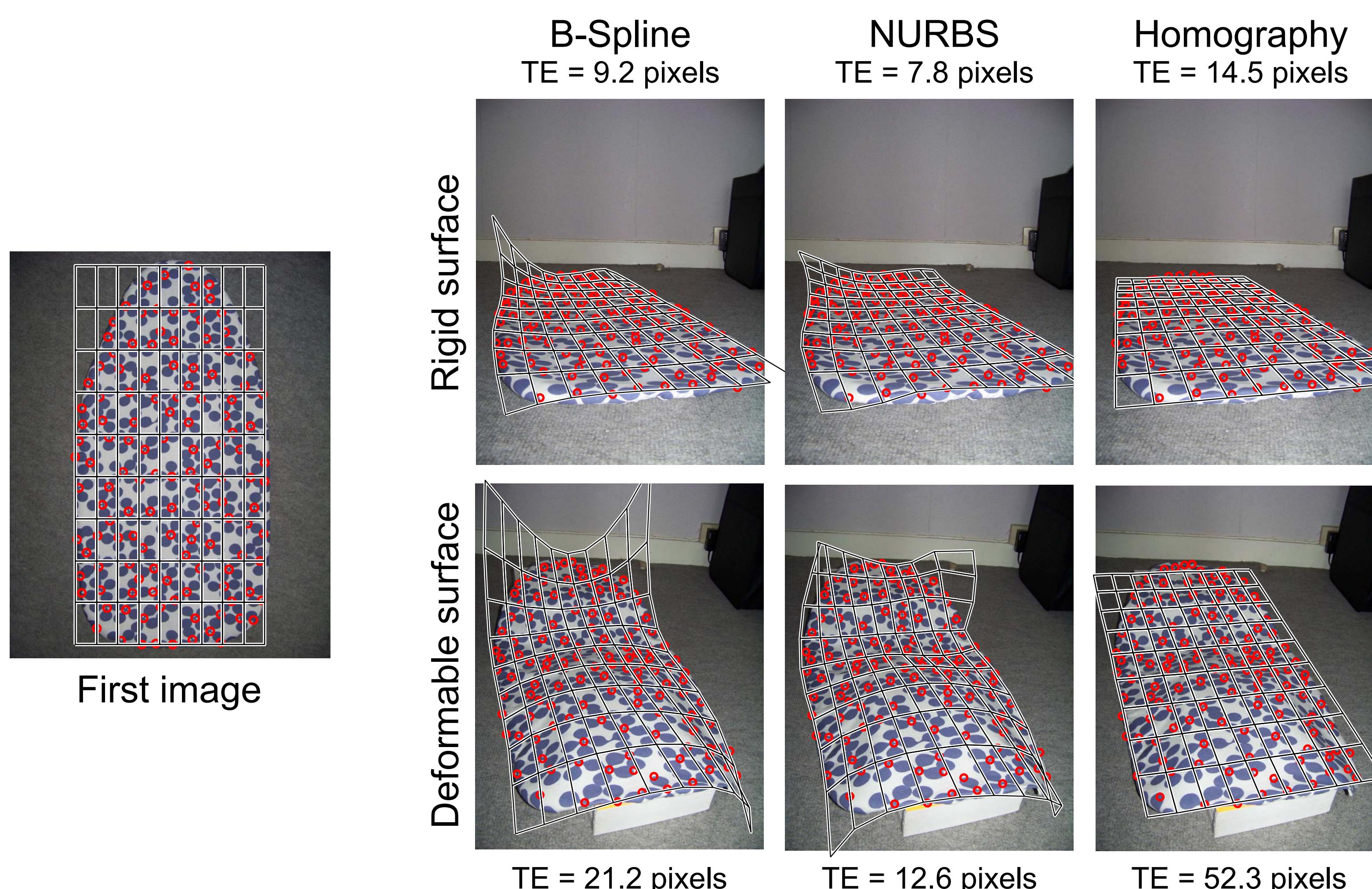
## Parameter Estimation

- Estimation of the parameters from point correspondences  $\mathbf{q}_k \leftrightarrow \mathbf{q}'_k$
- Minimization of the transfer error:

$$\min_{\mathbf{x}} \sum_{k=1}^r d^2(\mathbf{q}'_k, \mathcal{W}(\mathbf{q}_k; \mathbf{x}))$$

- B-Splines: Linear Least Squares
- NURBS: Non Linear Least Squares [2]
  - Iterative algorithm (e.g. Levenberg Marquardt)
  - Initialization:
    - Use an homography
    - Algebraic approximation to the transfer error
    - Use a B-Spline (best approach)

## Experiments / Illustrations



## Bibliography

- [1] D. Rueckert et al., Nonrigid registration using Free-Form Deformations: Applications to breast MR images, IEEE Transactions on Medical Imaging, 18(8):712-721, 1999
- [2] N. Carlson, NURBS surface fitting with Gauss-Newton, Lulea University of Technology, 2009.
- [3] F.L. Bookstein, Principal warps: Thin-Plate Splines and the decomposition of deformations, IEEE Transactions on Pattern Analysis and Machine Intelligence, 11(6):567-585, 1989