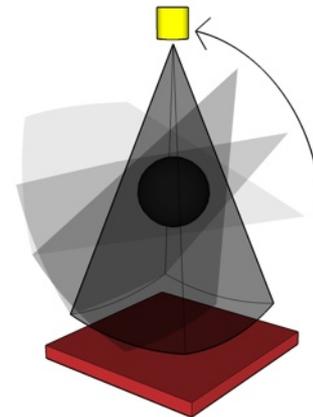


DA Final: Symbolic 3D+t Reconstruction From Cone-Beam Projections

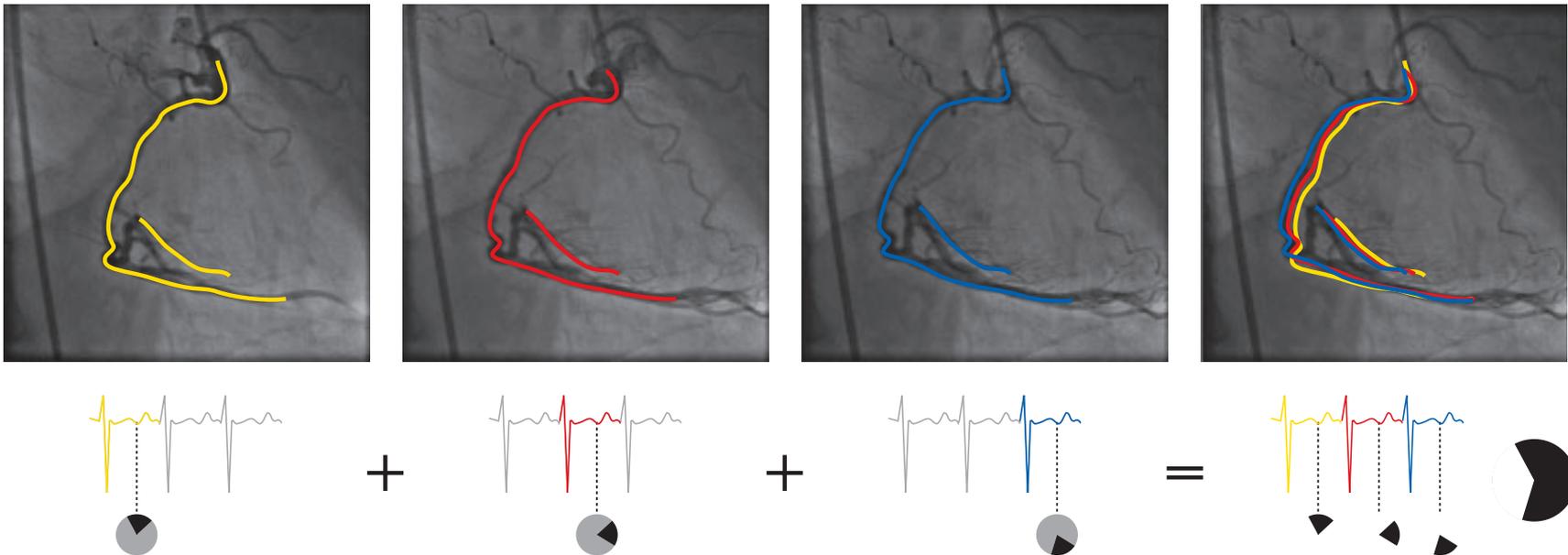
Jakob Vogel
(Supervised by Andreas Keil)

Background

- Reconstruct the heart (coronary arteries) from cone-beam projections
- Acquisition circumstances require to consider heart beat
- State of the art approaches use (retrospective) gating and treat the filtered image sequences as quasi-static [Blondel et al., Reconstruction of Coronary Arteries, 2006], ...
- Anatomical assumptions are usually not entirely valid

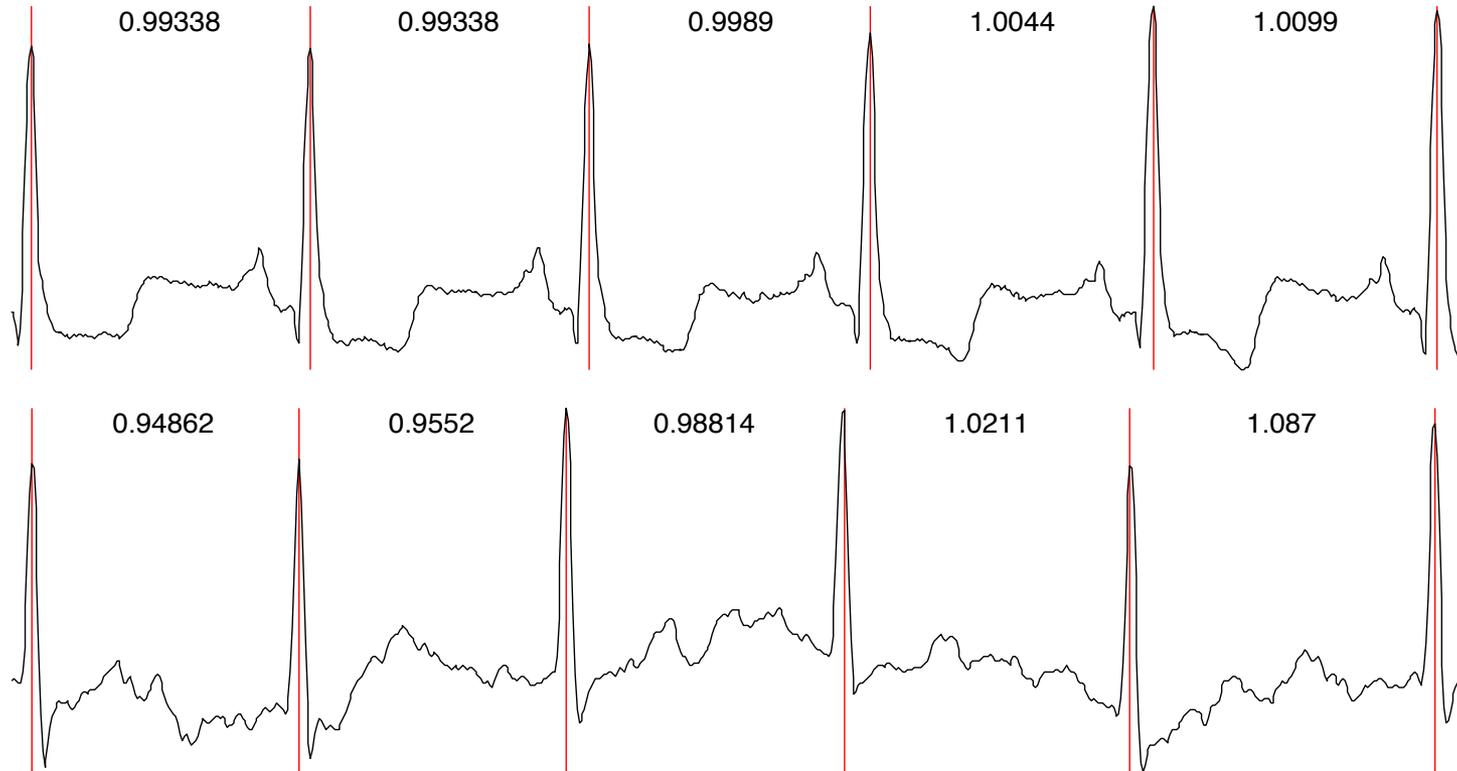


Background



Courtesy of Prof. Dr. Achenbach, UK Erlangen

Background

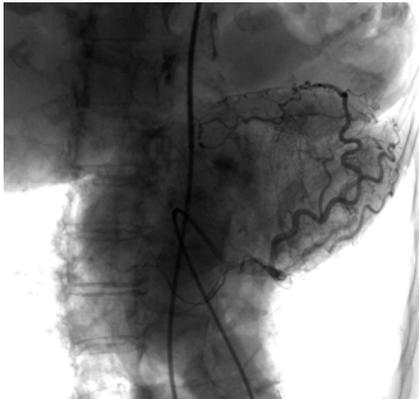


Data provided by Dr. Lauritsch, Siemens Healthcare

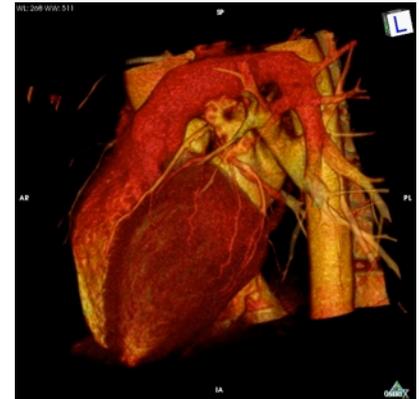
Concept

- *Is it possible to design an algorithm not using such assumptions?*
 - Simultaneously recover shape and motion
 - Simplify reconstruction
- Symbolic reconstruction
 - Vessel segmentation yields “likelihood” or “vesselness” images
 - Dynamic reconstruction computes a “likelihood” model consisting of
 - a static spatial model and
 - deformation information
- Results can be used for tomographic reconstruction

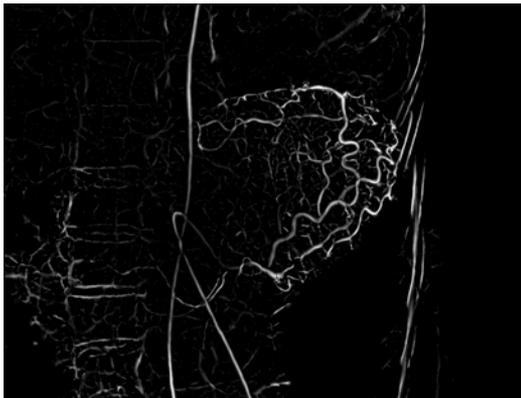
Concept



Tomographic Reconstruction

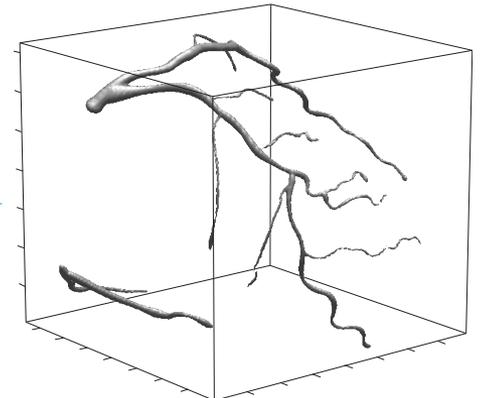


Calibration Data



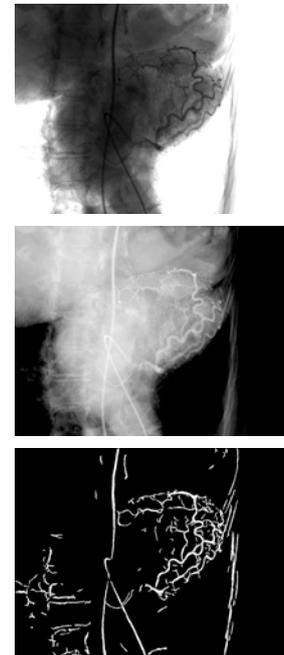
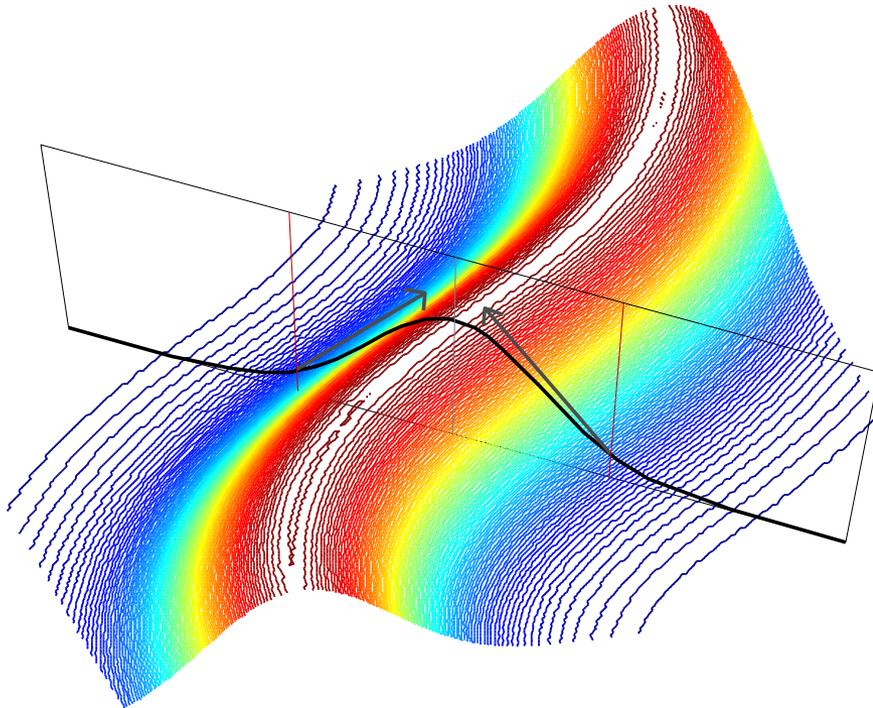
Symbolic Reconstruction

Deformation Model

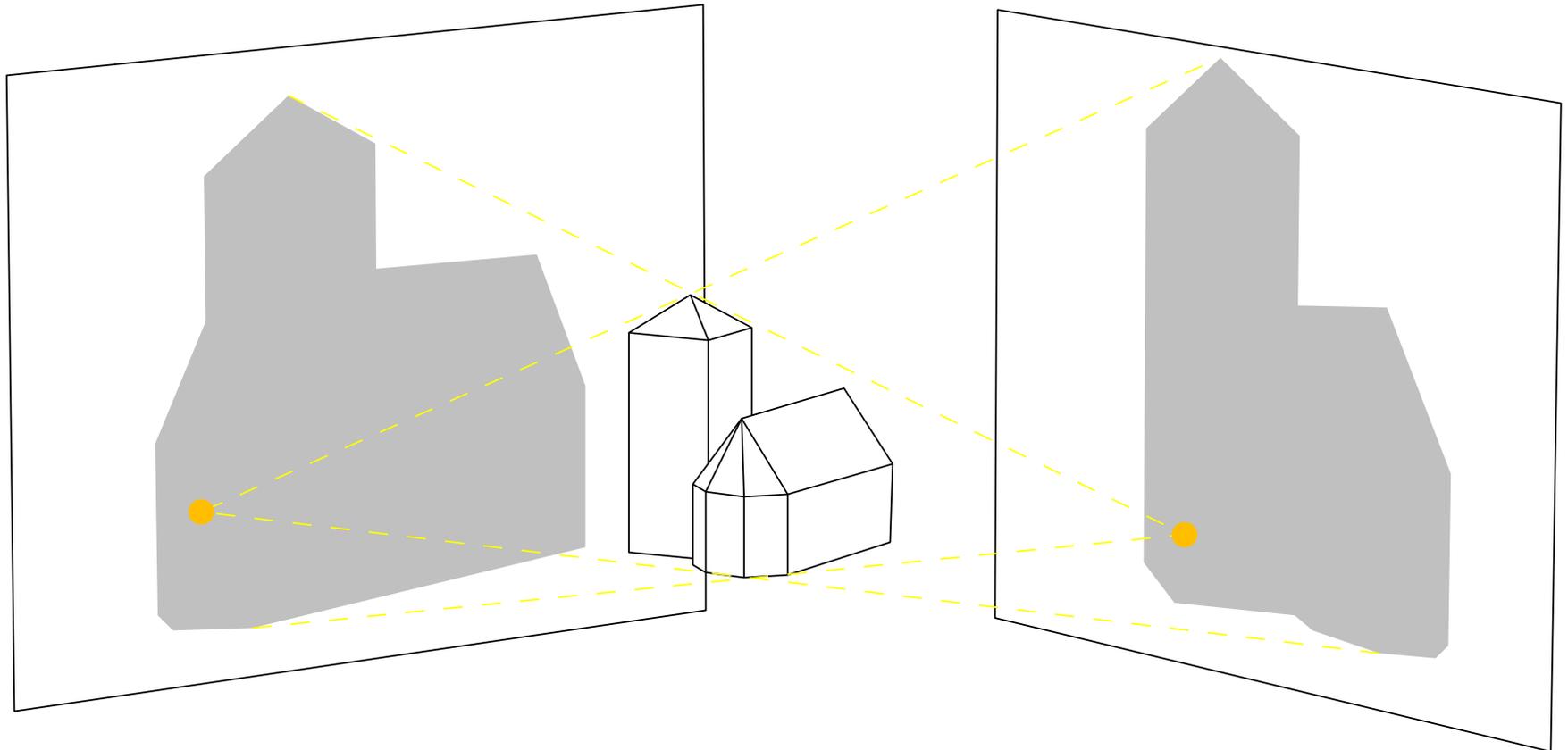


Vessel Segmentation

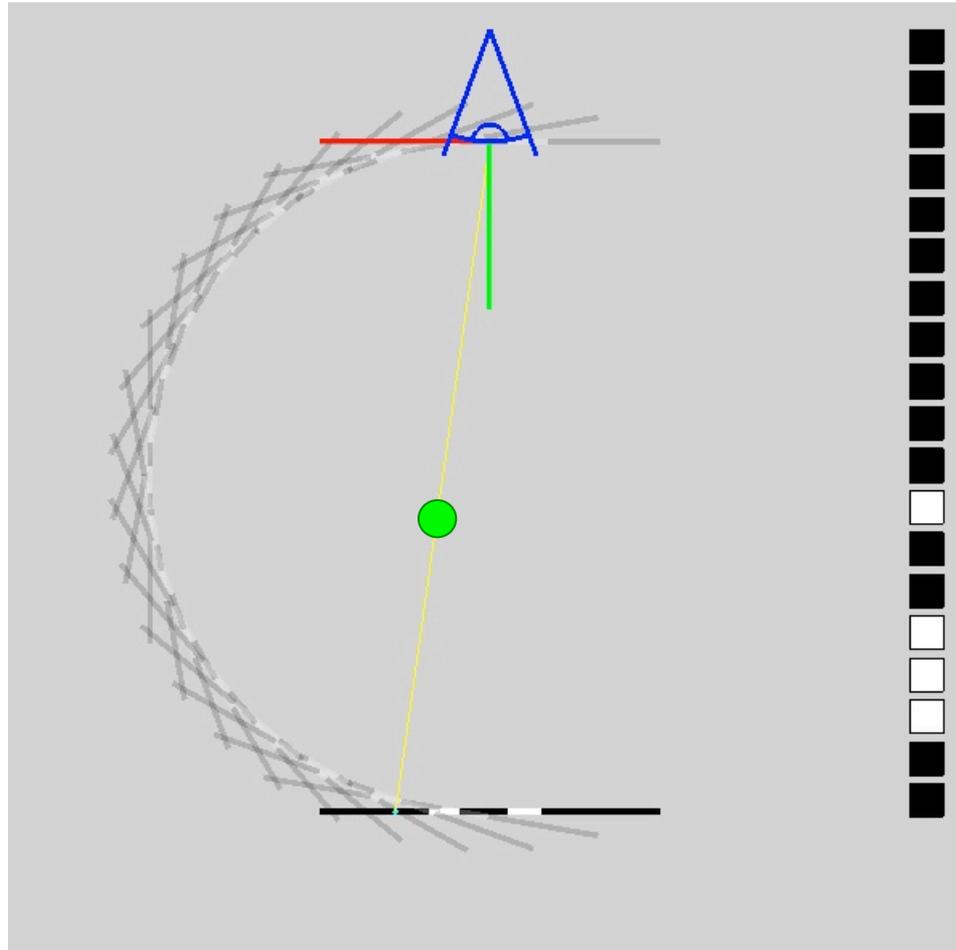
- IDP by Titus Rosu
- Multi-scale method based on [Koller et al., Multiscale Detection, 1995] and [Blondel 2006]



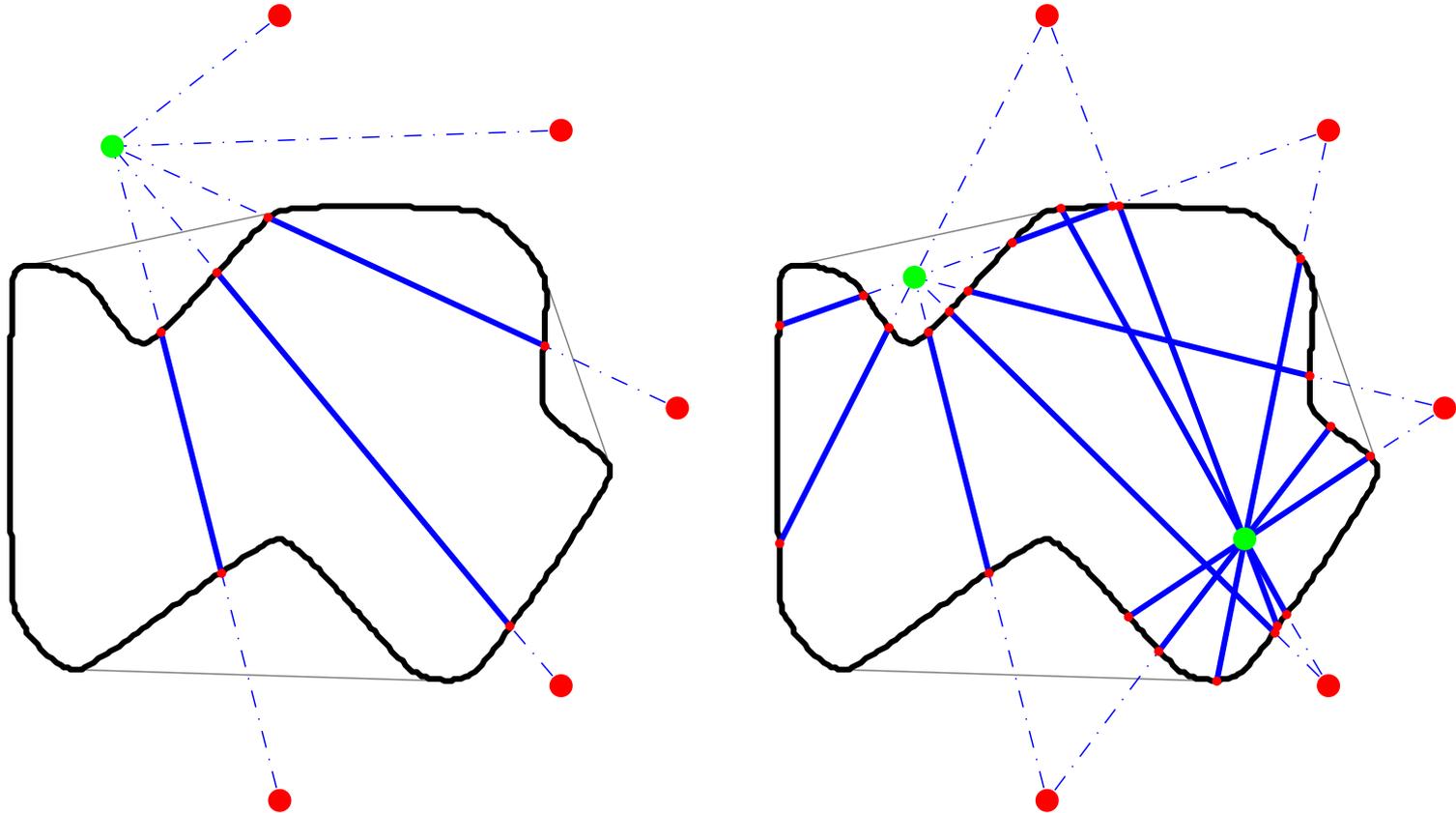
Shape from Silhouette



Shape from Silhouette



Shape from Silhouette



SFS reconstruction = segmentation of the reconstruction space

Level Sets

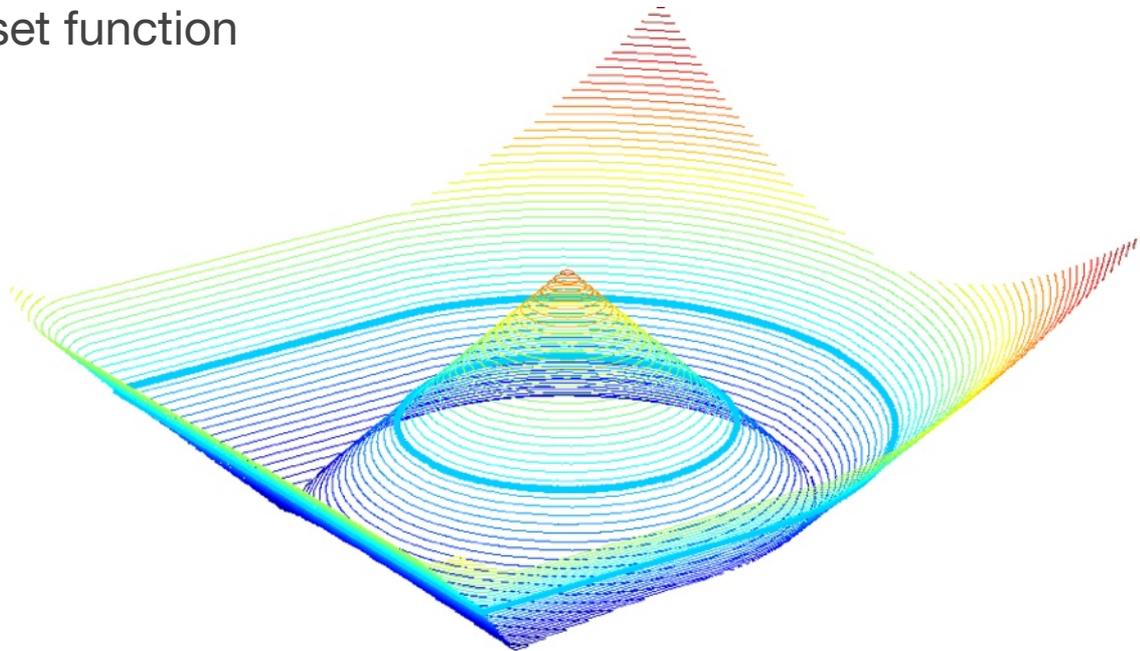
- Segmentation framework supporting extended mathematics [Sethian, Level Set Methods, 1999]
- Implicit model using a level set function

$$\Phi : \begin{cases} \Omega \subset \mathbf{R}^n & \rightarrow \mathbf{R} \\ \mathbf{x} & \mapsto \Phi(\mathbf{x}) \end{cases}$$

with the properties

$$\Phi(\mathbf{x}) \begin{cases} < 0 & : \mathbf{x} \text{ is inside } \Gamma \\ = 0 & : \mathbf{x} \in \Gamma \\ > 0 & : \mathbf{x} \text{ is outside } \Gamma \end{cases}$$

- Signed distance constraint



Level Sets

- *Wave front approach:* The zero contour is regarded as wave front and forces are defined driving the contour towards the desired position
- *Variational approach:* An energy functional punishing false positives and false negatives can be used to derive a PDE [Chan et al., Active Contours without Edges, 2001]

Variational Level Sets

- Set up an energy functional depending on the level set function

$$E(\Phi) = \int_{\Omega} I(\mathbf{x}, \Phi(\mathbf{x}), \dots) \, d\mathbf{x}$$

- Calculate the derivative of the functional with respect to the level set function

$$\frac{dE}{d\Phi}(\Phi; \eta) = \int_{\Omega} I_{\Phi}(\mathbf{x}, \Phi(\mathbf{x}), \dots) \cdot \eta(\mathbf{x}) \, d\mathbf{x}$$

- An optimal segmentation over artificial time is given as solution to a PDE

$$\Phi_{\tau}(\mathbf{x}) = -I_{\Phi}(\mathbf{x}, \Phi(\mathbf{x}), \dots) \quad \forall \mathbf{x} \in \Omega$$

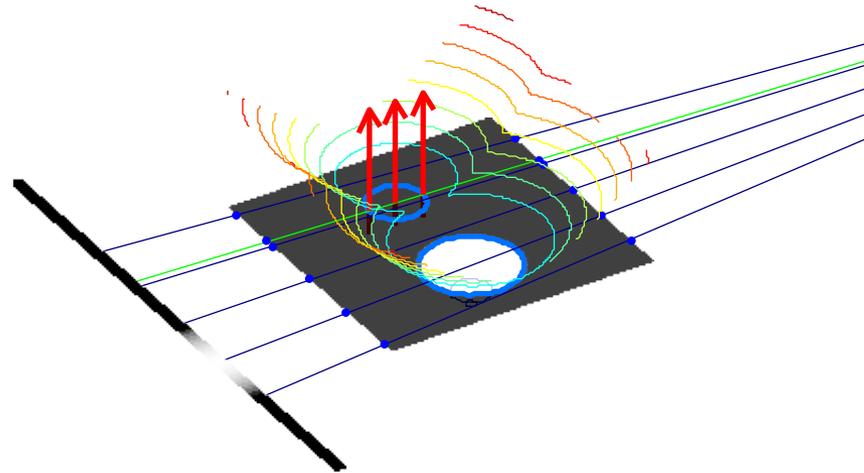
Level Sets

- The voxel-wise error term weights false segmentations and may contain additional regularization expressions
- Design is often using the Heaviside function

$$H(x) = \begin{cases} 0 & : x < 0 \\ \frac{1}{2} & : x = 0 \\ 1 & : x > 0 \end{cases}$$

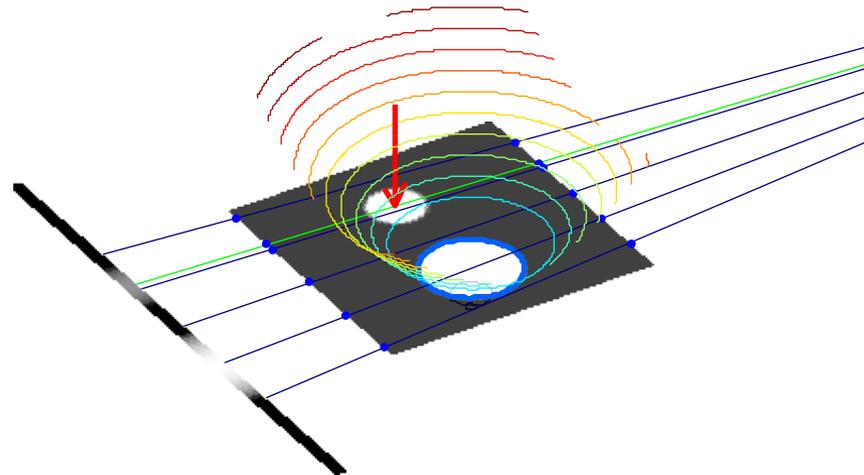
Error Terms

$$\begin{aligned}
 E_{\text{FP}}(\Phi) &= \sum_{i=1}^n \int_{\Omega_V} \overset{\text{false}}{S_{\text{FP}}(\mathbf{X})} \cdot \overset{\text{positive}}{S_{\text{P}}(\mathbf{X})} \cdot \overset{\text{penalty}}{p_{\text{FP}}(\mathbf{X})} d\mathbf{X} \\
 &= \sum_{i=1}^n \int_{\Omega_V} \overset{\text{false}}{H\left(\frac{1}{2} - I_i(P_i(\mathbf{X}))\right)} \cdot \overset{\text{positive}}{[1 - H(\Phi(\mathbf{X}))]} \cdot \overset{\text{penalty}}{[1 - I_i(P_i(\mathbf{X}))]} d\mathbf{X}
 \end{aligned}$$



Error Terms

$$\begin{aligned}
 E_{\text{FN}}(\Phi) &= \sum_{i=1}^n \int_{\Omega_I} \overset{\text{false}}{S_{\text{FN}}(\mathbf{x})} \cdot \overset{\text{negative}}{S_N(\mathbf{x})} \cdot \overset{\text{penalty}}{p_{\text{FN}}(\mathbf{x})} d\mathbf{x} \\
 &= \sum_{i=1}^n \int_{\Omega_I} \overset{\text{false}}{H\left(I_i(\mathbf{x}) - \frac{1}{2}\right)} \cdot \overset{\text{negative}}{H\left(\min_{\mathbf{X} \in P_i^{-1}(\mathbf{x})} \Phi(\mathbf{X})\right)} \cdot \overset{\text{penalty}}{I_i(\mathbf{x})} d\mathbf{x}
 \end{aligned}$$



Remarks

- Build a composite error term
 - Different integration domains require to add weights
 - Shape regularization enforces a smooth surface

$$E(\Phi_0, \alpha) = \lambda_{\text{FN}} \cdot E_{\text{FN}}(\Phi_0, \alpha) + \lambda_{\text{FP}} \cdot E_{\text{FP}}(\Phi_0, \alpha) + \lambda_{\text{shape}} \cdot E_{\text{shape}}(\Phi_0)$$

- Differentiation yields update terms for numerical implementation
 - Voxel-wise evolution of a discrete level set function
 - Implementation of the FN updates requires a “hack”
- Separate “reinitialization” guarantees signed distance constraint

Submerged Aquatic Vegetation

of the Chesapeake Bay

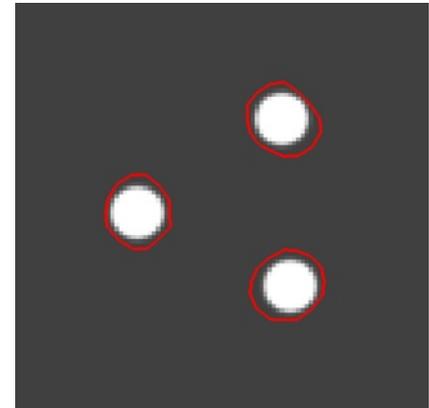
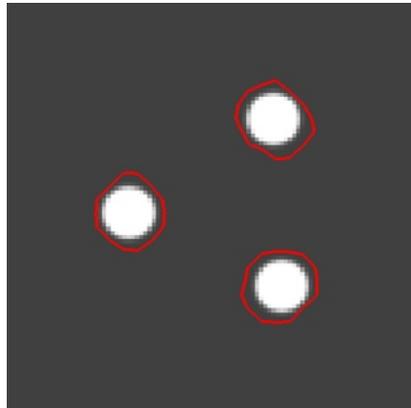
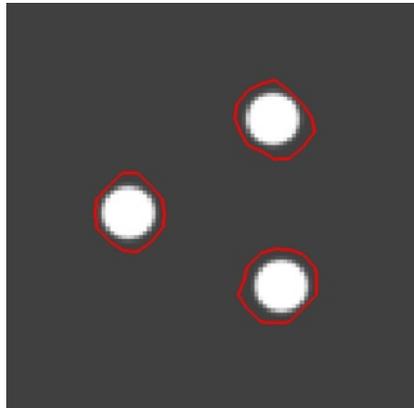
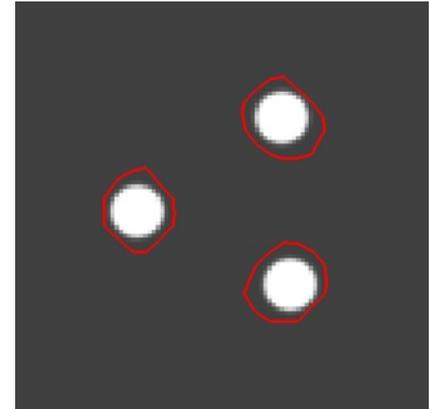
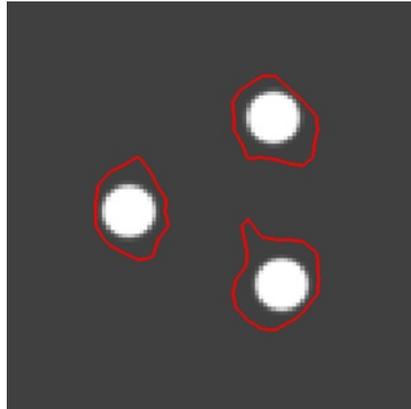
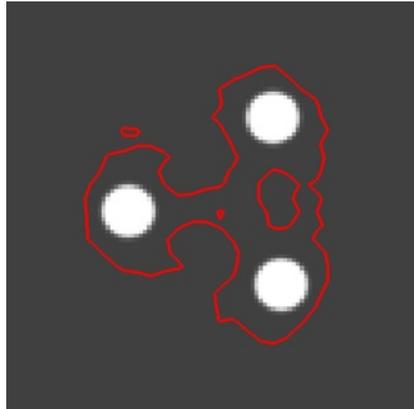
— *Proceedings of the Chesapeake Bay Conference* —

EDITED BY
JAMES H. COOPER, JR., *University of Maryland System*

Published by
THE UNIVERSITY OF MARYLAND SYSTEM
College Park, Maryland

1974
Pp. 300
\$12.50

Results



Dynamic Level Sets

- A 3D+t problem could be modeled with a 4D level set function, but this approach would require extensive regularization

- Instead, the motion is modeled with a time-dependent mapping

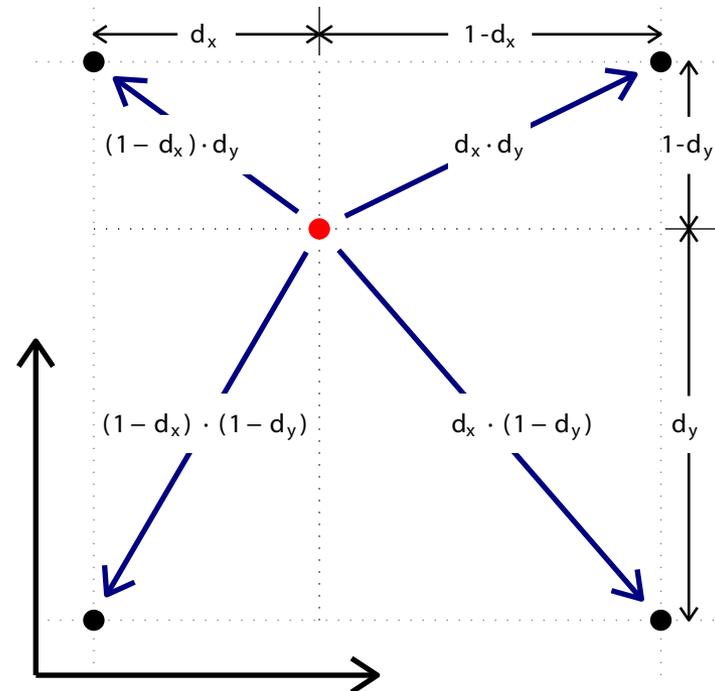
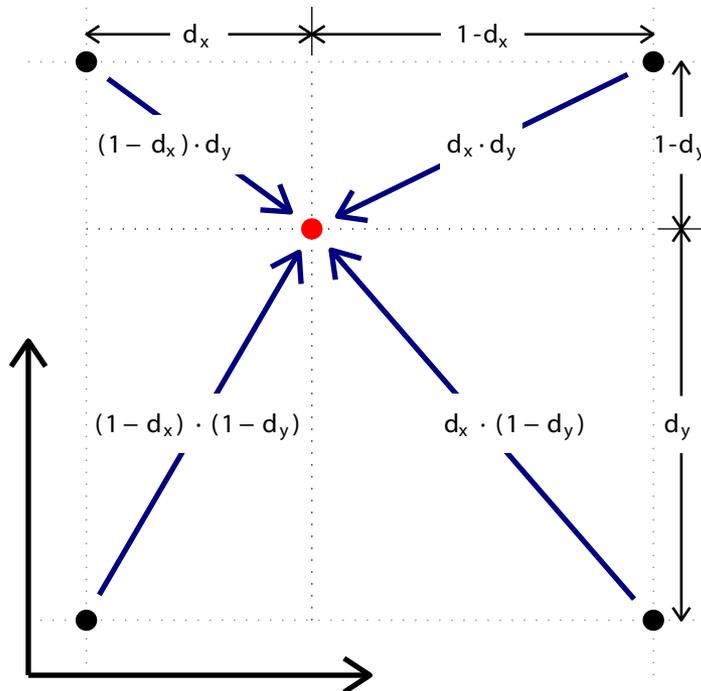
$$\varphi(\mathbf{x}, t, \alpha) = \mathbf{R}(t, \alpha) \cdot \mathbf{x} + \mathbf{T}(t, \alpha) + \mathbf{u}(\mathbf{x}, t, \alpha)$$

- A 4D level set function is then emulated using this mapping – and thus implicitly regularized – as

$$\Phi(\mathbf{x}, t, \alpha) = \Phi_0(\varphi(\mathbf{x}, t, \alpha))$$

Dynamic Level Sets

- Two extensions to the static versions:
 - Every access to the level set function needs to be “deformed”
 - The deformation model needs to be updated



Dynamic Level Sets

- Evolve the motion field along with the level set function over artificial time

$$\frac{d\alpha}{d\tau} = -\lambda_{\text{motion}} \frac{dE}{d\alpha}$$

- The derivative computes as product of several other derivatives

$$\frac{dE}{d\alpha} = \frac{dE}{d\Phi_0} \cdot \nabla_{\mathbf{x}} \Phi_0 \cdot \frac{d\varphi}{d\alpha}$$

- Interleave the algorithms to run both optimizations *simultaneously*

Algorithm

initialize *deformation* to identity, *shape* to unknown

until convergence [artificial time]

for all discrete nodes of the reconstruction volume

for all discrete times

 reconstruct *shape* considering *deformation*

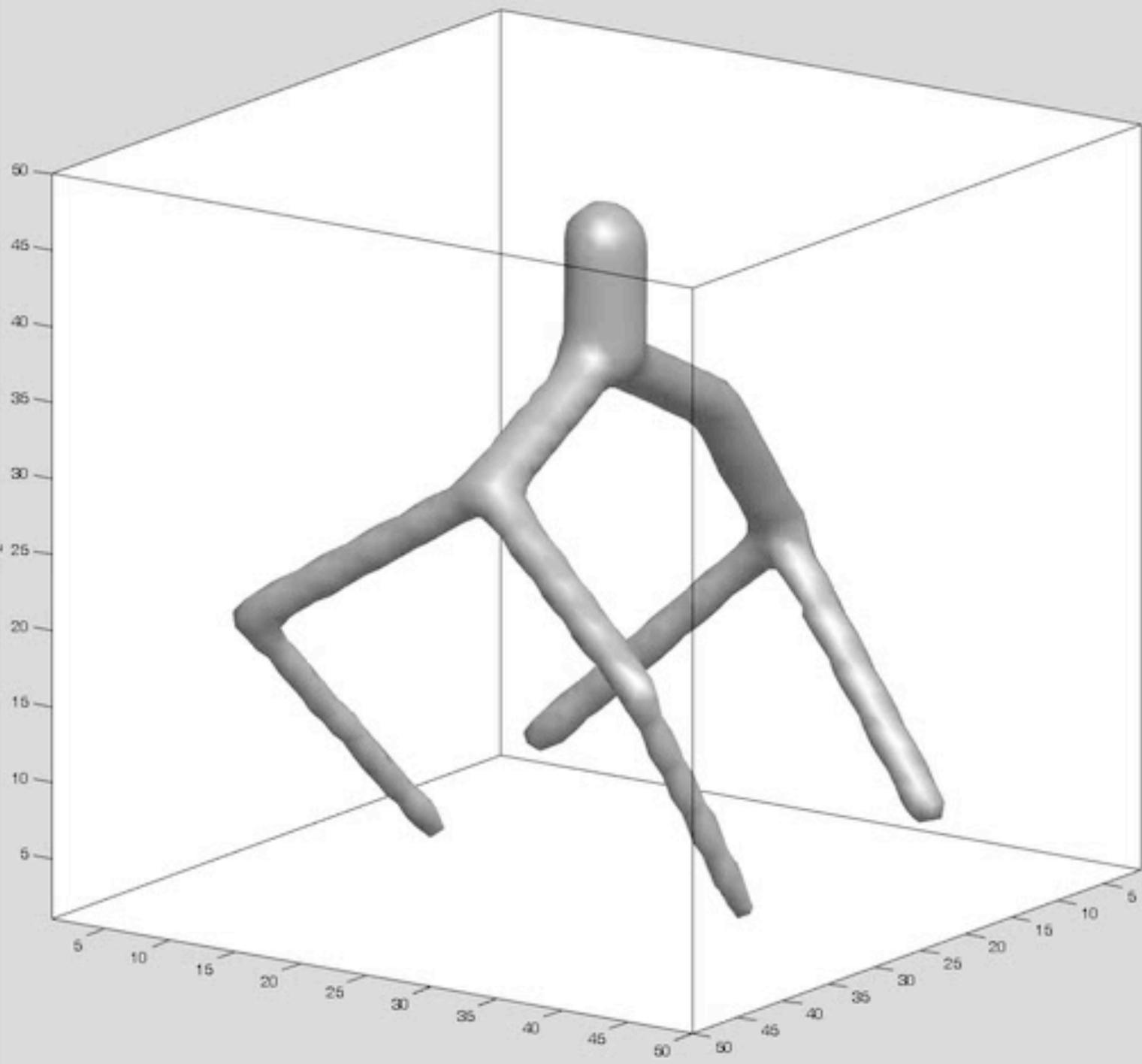
 update *deformation* for the current node using gradient descent

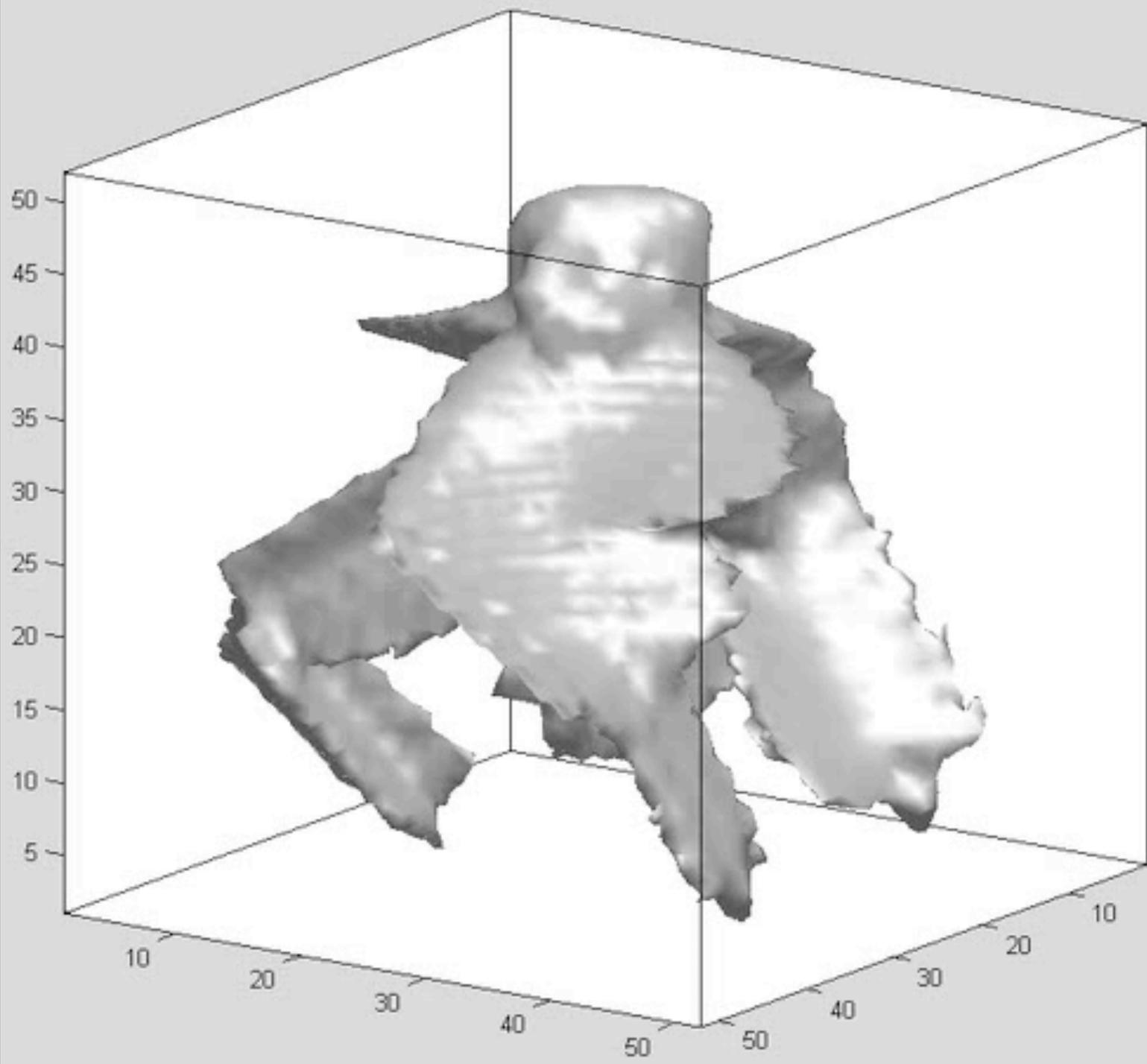
end

end

end

use models for tomographic reconstruction, diagnosis, and navigation





Experiments

- Method works for restricted motion models using phantom data
 - 100 iterations on down-sampled data take about 1 hour on a 24 core computer
 - Reconstruction volume has 50^3 voxels at a 3 mm spacing

Experiments – Rigid Motion

Data Set	Noise Level	Mean	Standard Deviation	Maximum	Median
Synthetic	0%	0.54	0.30	2.19	0.47
Synthetic	25%	0.68	0.36	3.14	0.60
Synthetic	50%	2.36	2.53	11.73	1.18
Phantom	0%	0.91	0.48	4.41	0.82
Phantom	25%	0.88	0.46	4.37	0.81
Phantom	50%	4.15	2.70	9.79	3.86

$$\epsilon_{k,i} = \left\| [\hat{\mathbf{R}}(t_i) \cdot \mathbf{X}_k + \hat{\mathbf{T}}(t_i)] - [\mathbf{R}(\boldsymbol{\alpha}, t_i) \cdot \mathbf{X}_k + \mathbf{T}(\boldsymbol{\alpha}, t_i)] \right\|_2$$

Experiments – Deformable Motion

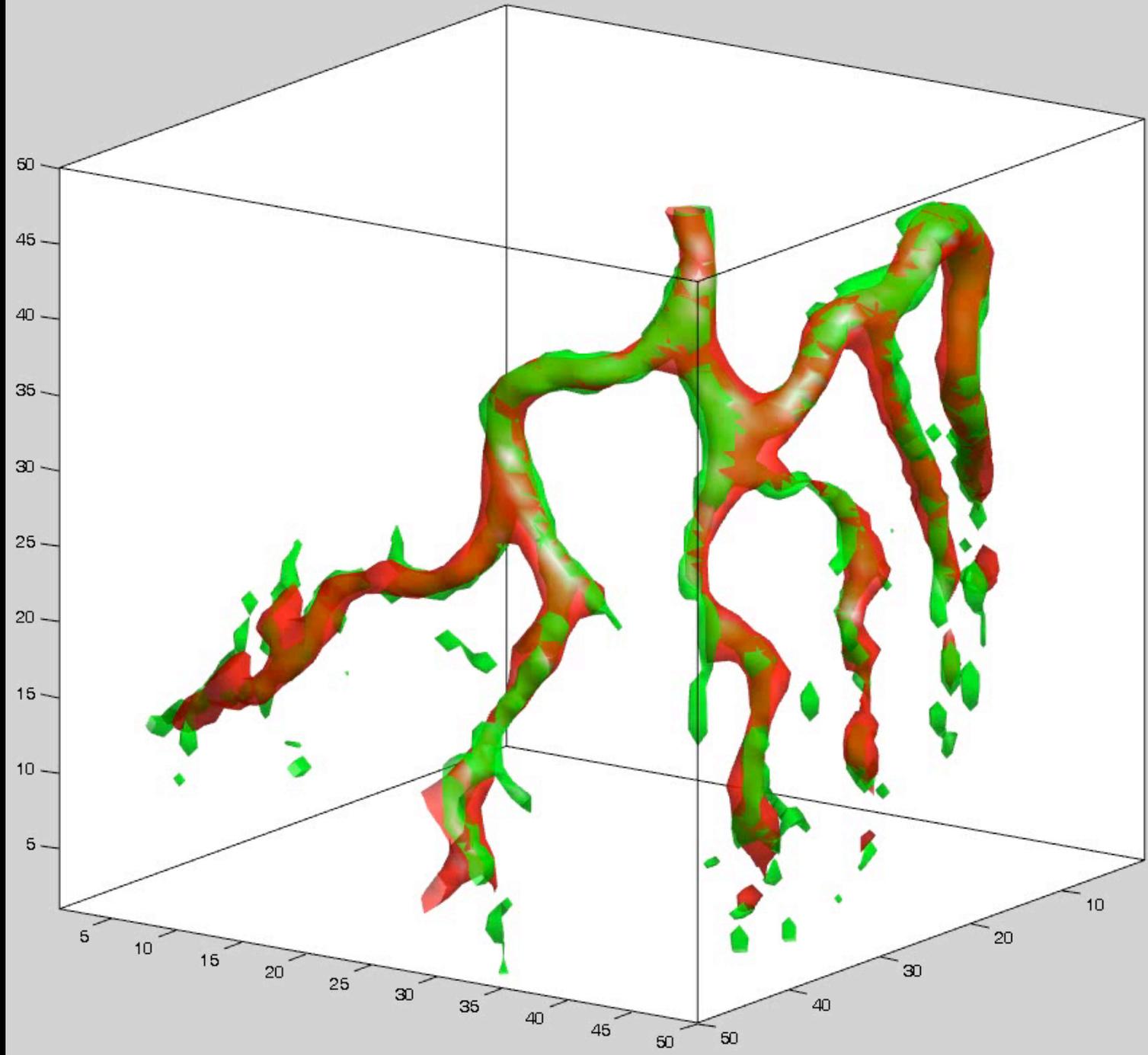
Data Set	Noise Level	Overlap Ratio	Sensitivity	Specificity
Synthetic	0%	85.1%	86.1%	99.9%
Synthetic	10%	84.9%	84.4%	99.9%
Synthetic	20%	84.6%	83.5%	99.9%
Synthetic	30%	83.8%	80.1%	99.9%
Synthetic	40%	83.2%	80.1%	99.9%
Synthetic	50%	81.3%	75.9%	99.9%
Phantom	0%	66.7%	75.2%	99.6%
Phantom	10%	66.6%	78.0%	99.6%
Phantom	20%	65.0%	73.8%	99.6%
Phantom	30%	67.0%	74.2%	99.6%
Phantom	40%	66.3%	72.8%	99.6%
Phantom	50%	64.7%	71.7%	99.6%

Conclusion

- Only limited sensibility to noise
- Higher resolutions, more speed, optimal motion model
- GPUs?
 - Portions could be ported right away
 - Level Set implementation requires random write access
- Tests with realistic phantom [XCAT] and real data



Time: 0.000



Thank you!

Numerical Realization

- Approximate the derivative of the level set function using a forward difference operator

$$\Phi_{\tau}(\mathbf{x}) \approx \frac{\Phi[\tau + \Delta\tau](\mathbf{x}) - \Phi[\tau](\mathbf{x})}{\Delta\tau}$$

- Use the PDE to write an update formula using artificial time steps

$$\begin{aligned}\Phi[\tau + \Delta\tau](\mathbf{x}) &\approx \Phi[\tau](\mathbf{x}) + \Delta\tau \cdot \Phi_{\tau}(\mathbf{x}) \\ &= \Phi[\tau](\mathbf{x}) - \Delta\tau \cdot I_{\Phi}(\Phi(\mathbf{x}))\end{aligned}$$

Energy Term

- Shape regularization

$$E_{\text{shape}}(\Phi_0) = \int_{V_0} \delta(\Phi_0(\mathbf{x})) \cdot \|\nabla\Phi_0(\mathbf{x})\| \, d\mathbf{x}$$

Reconstruction Errors

- Overlap ratio

$$R = \frac{2 \cdot T_p}{(T_p + F_n) + (T_p + F_p)}$$

- Sensitivity

$$S_e = \frac{T_p}{T_p + F_n}$$

- Specificity

$$S_p = \frac{T_n}{T_n + F_p}$$

- All these equations use voxel counts and depend on resolution hence